SOLUTION.
Use TACTIC 2. Choose appropriate numbers. Assume Delphine can type 1 page per hour and Eliane can type 2. Assume Eliane charges $1.00 per page and Delphine charges $1.50. Then in 9 hours, Eliane types 18 pages, earning **$18.00**. In 12 hours, Delphine types 12 pages, earning 12 × $1.50 = **$18.00**. The answer is **C**.

TACTIC

3 Make the Problem Easier: Do the Same Thing to Each Quantity

A quantitative comparison question can be treated as an equation or an inequality. Either:

Quantity A < Quantity B, or
Quantity A = Quantity B, or
Quantity A > Quantity B

In solving an equation or an inequality, you can always add the same thing to each side or subtract the same thing from each side. Similarly, in solving a quantitative comparison, you can always add the same thing to quantities A and B or subtract the same thing from quantities A and B. You can also multiply or divide each side of an equation or inequality by the same number, *but in the case of inequalities you can do this only if the number is positive.* Since you don't know whether the quantities are equal or unequal, you cannot multiply or divide by a variable *unless you know that it is positive.* If quantities A and B are both positive you may square them or take their square roots.

To illustrate the proper use of TACTIC 3, we will give alternative solutions to examples 4, 5, and 6, which we already solved using TACTIC 1.

EXAMPLE 4

$m > 0$ and $m \neq 1$

Quantity A	Quantity B
m^2	m^3

SOLUTION.

	Quantity A	Quantity B
Divide each quantity by m^2 (that's OK — m^2 is positive):	$\frac{m^2}{m^2} = 1$	$\frac{m^3}{m^2} = m$

This is a much easier comparison. Which is greater, m or 1? We don't know. We know $m > 0$ and $m \neq 1$, but it could be greater than 1 or less than 1. The answer is **D**.

EXAMPLE 5

Quantity A	Quantity B
$13y$	$15y$

SOLUTION.

	Quantity A	Quantity B
Subtract $13y$ from each quantity:	$13y - 13y = 0$	$15y - 13y = 2y$

Since there are no restrictions on y, $2y$ could be greater than, less than, or equal to 0. The answer is **D**.

EXAMPLE 6

Quantity A	Quantity B
$w + 11$	$w - 11$

SOLUTION.

	Quantity A	Quantity B
Subtract w from each quantity:	$(w + 11) - w = 11$	$(w - 11) - w = -11$

Clearly, 11 is greater than −11. Quantity **A** is greater.

Here are five more examples on which to practice TACTIC 3.

EXAMPLE 13

Quantity A	Quantity B
$\frac{1}{3}+\frac{1}{4}+\frac{1}{9}$	$\frac{1}{9}+\frac{1}{3}+\frac{1}{5}$

SOLUTION.

	Quantity A	Quantity B
Subtract $\frac{1}{3}$ and $\frac{1}{9}$ from each quantity:	$\cancel{\frac{1}{3}}+\frac{1}{4}+\cancel{\frac{1}{9}}$	$\cancel{\frac{1}{9}}+\cancel{\frac{1}{3}}+\frac{1}{5}$

Since $\frac{1}{4} > \frac{1}{5}$, the answer is **A**.

EXAMPLE 14

Quantity A	Quantity B
$(43 + 59)(17 - 6)$	$(43 + 59)(17 + 6)$

SOLUTION.

	Quantity A	Quantity B
Divide each quantity by (43 + 59):	~~(43 + 59)~~(17 – 6)	~~(43 + 59)~~(17 + 6)

Clearly, (17 + 6) > (17 – 6). The answer is **B**.

EXAMPLE 15

Quantity A	Quantity B
(43 – 59)(43 – 49)	(43 – 59)(43 + 49)

SOLUTION.

> **CAUTION**
>
> (43 – 59) is negative, and you may not divide the two quantities by a negative number.

The easiest alternative is to note that Quantity A, being the product of 2 negative numbers, is positive, whereas Quantity B, being the product of a negative number and a positive number, is negative, and so Quantity A is greater.

EXAMPLE 16

a is a negative number

Quantity A	Quantity B
a^2	$-a^2$

SOLUTION.

	Quantity A	Quantity B
Add a^2 to each quantity:	$a^2 + a^2 = 2a^2$	$-a^2 + a^2 = 0$

Since a is negative, $2a^2$ is positive. The answer is **A**.

EXAMPLE 17

Quantity A	Quantity B
$\frac{\sqrt{20}}{2}$	$\frac{5}{\sqrt{5}}$

SOLUTION.

	Quantity A	Quantity B
Square each quantity:	$\left(\frac{\sqrt{20}}{2}\right)^2 = \frac{20}{4} = 5$	$\left(\frac{5}{\sqrt{5}}\right)^2 = \frac{25}{5} = 5$

The answer is **C**.

TACTIC 4

Ask "Could They Be Equal?" and "Must They Be Equal?"

TACTIC 4 has many applications, but is most useful when one of the quantities contains a variable and the other contains a number. In this situation ask yourself, "Could they be equal?" If the answer is "yes," eliminate A and B, and then ask, "Must they be equal?" If the second answer is "yes," then C is correct; if the second answer is "no," then choose D. When the answer to "Could they be equal?" is "no," we usually know right away what the correct answer is. In both questions, "Could they be equal" and "Must they be equal," the word *they* refers, of course, to quantities A and B.

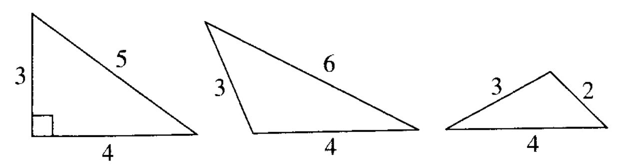
Let's look at a few examples.

EXAMPLE 18

The sides of a triangle are 3, 4, and x

Quantity A	Quantity B
x	5

SOLUTION.
Could they be equal? Could $x = 5$? Of course. That's the all-important 3-4-5 right triangle. Eliminate A and B. Must they be equal? Must $x = 5$? If you're not sure, try drawing an acute or an obtuse triangle. The answer is No. Actually, x can be any number satisfying: $1 < x < 7$. (See KEY FACT J12, the triangle inequality, and the figure below.) The answer is **D**.



EXAMPLE 19

$56 < 5c < 64$

Quantity A	Quantity B
c	12

SOLUTION.
Could they be equal? Could $c = 12$? If $c = 12$, then $5c = 60$, so, yes, they could be equal. Eliminate A and B. Must they be equal? Must $c = 12$? Could c be more or less than 12? BE CAREFUL: 5 × 11 = 55, which is too small; and 5 × 13 = 65, which is too big. Therefore, the only *integer* that c could be is 12; but *c doesn't have to be an integer*. The *only* restriction is that $56 < 5c < 64$. If $5c$ were 58 or 61.6 or 63, then c would not be 12. The answer is **D**.

EXAMPLE 20

School A has 100 teachers and School B has 200 teachers. Each school has more female teachers than male teachers.

Quantity A	Quantity B
The number of female teachers at School A	The number of female teachers at School B

SOLUTION.
Could they be equal? Could the number of female teachers be the same in both schools? No. More than half (i.e., more than 100) of School B's 200 teachers are female, but School A has only 100 teachers in all. The answer is **B**.

EXAMPLE 21

$$(m + 1)(m + 2)(m + 3) = 720$$

Quantity A	Quantity B
$m + 2$	10

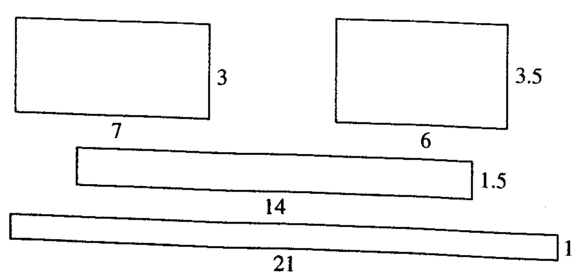
SOLUTION.
Could they be equal? Could $m + 2 = 10$? No, if $m + 2 = 10$, then $m + 1 = 9$ and $m + 3 = 11$, and $9 \times 10 \times 11 = 990$, which is too big. The answer is *not* C, and since $m + 2$ clearly has to be smaller than 10, the answer is **B**.

EXAMPLE 22

Quantity A	Quantity B
The perimeter of a rectangle whose area is 21	20

SOLUTION.
Could they be equal? Could a rectangle whose area is 21 have a perimeter of 20? Yes, if its length is 7 and its width is 3: $7 + 3 + 7 + 3 = 20$. Eliminate A and B. Must they be equal? If you're *sure* that there is no other rectangle with an area of 21, then choose C; if you're *not* sure, guess between C and D; if you *know* there are other rectangles of area 21, choose D.

There are other possibilities — lots of them; here are a 7×3 rectangle and a few other rectangles whose areas are 21:



TACTIC 5 Don't Calculate: Compare

Avoid unnecessary calculations. You don't have to determine the exact values of Quantity A and Quantity B; you just have to compare them.

TACTIC 5 is the special application of TACTIC 7, Chapter 10 (Don't do more than you have to) to quantitative comparison questions. Using TACTIC 5 allows you to solve many quantitative comparisons without doing tedious calculations, thereby saving you valuable test time that you can use on other questions. *Before you start calculating,* stop, look at the quantities, and ask yourself, "Can I easily and quickly determine which quantity is greater without doing *any* arithmetic?" Consider Examples 23 and 24, which look very similar, but really aren't.

EXAMPLE 23

Quantity A	Quantity B
37×43	30×53

EXAMPLE 24

Quantity A	Quantity B
37×43	39×47

Example 23 is very easy. Just multiply: $37 \times 43 = 1591$ and $30 \times 53 = 1590$. The answer is **A**.

Example 24 is even easier. *Don't* multiply. In less time than it takes to do the multiplications, even with the calculator, you can see that $37 < 39$ and $43 < 47$, so clearly $37 \times 43 < 39 \times 47$. The answer is **B**. *You don't get any extra credit for taking the time to determine the value of each product!*

Remember: do not start calculating immediately. Always take a second or two to glance at each quantity. In Example 23 it's not at all clear which product is larger, so you have to multiply. In Example 24, however, no calculations are necessary.

These are problems on which poor test-takers do a lot of arithmetic and good test-takers think! Practicing TACTIC 5 will help you become a good test-taker.

Now, test your understanding of TACTIC 5 by solving these problems.

EXAMPLE 25

Quantity A	Quantity B
The number of years from 1776 to 1929	The number of years from 1767 to 1992

EXAMPLE 26

Quantity A	Quantity B
$45^2 + 25^2$	$(45 + 25)^2$

EXAMPLE 27

Quantity A	Quantity B
$45(35 + 65)$	$45 \times 35 + 45 \times 65$

EXAMPLE 28

Marianne earned a 75 on each of her first three math tests and an 80 on her fourth and fifth tests.

Quantity A	Quantity B
Marianne's average after 4 tests	Marianne's average after 5 tests

SOLUTIONS 25–28

Performing the Indicated Calculations	Using TACTIC 5 to Avoid Doing the Calculations
25. Quantity A: $1929 - 1776 = 153$ Quantity B: $1992 - 1767 = 225$ The answer is **B**.	25. The subtraction is easy enough, but why do it? The dates in Quantity **B** start earlier and end later. Clearly, they span more years. You don't need to know how many years. The answer is **B**.
26. Quantity A: $45^2 + 25^2 = 2025 + 625 = 2650$ Quantity B: $(45 + 25)^2 = 70^2 = 4900$ The answer is **B**.	26. For *any positive numbers a* and *b*: $(a + b)^2 > a^2 + b^2$. You should do the calculations only if you don't know this fact. The answer is **B**.
27. Quantity A: $45(35 + 65) = 45(100) = 4500$ Quantity B: $45 \times 35 + 45 \times 65 = 1575 + 2925 = 4500$ The answer is **C**.	27. This is just the distributive property (KEY FACT A20), which states that, for *any* numbers *a*, *b*, *c*: $a(b + c) = ab + ac$. The answer is **C**.
28. Quantity A: $\frac{75+75+75+80}{4} = \frac{305}{4} = 76.25$ Quantity B: $\frac{75+75+75+80+80}{5} = \frac{385}{5} = 77$ The answer is **B**.	28. Remember, you want to know which average is higher, *not* what the averages are. After 4 tests Marianne's average is clearly less than 80, so an 80 on the fifth test had to *raise* her average (KEY FACT E4). The answer is **B**.

Caution

TACTIC 5 is important, but *don't spend a lot of time looking for ways to avoid a simple calculation.*

TACTIC 6 Know When to Avoid Choice D

If Quantity A and Quantity B are both fixed numbers, the answer cannot be D.

Notice that D was not the correct answer to any of the six examples discussed under TACTIC 5. Those problems had no variables. The quantities were all specific numbers. In each of the next four examples, Quantity A and Quantity B are also fixed numbers. In each case, either the two numbers are equal or one is greater than the other. It can *always* be determined, and so D *cannot be the correct answer to any of these problems*. If, while taking the GRE, you find a problem of this type that you can't solve, just guess: A, B, or C. Now try these four examples.

EXAMPLE 29

Quantity A	Quantity B
The number of seconds in one day	The number of days in one century

EXAMPLE 30

Quantity A	Quantity B
The area of a square whose sides are 4	Twice the area of an equilateral triangle whose sides are 4

EXAMPLE 31

Three fair coins are flipped.

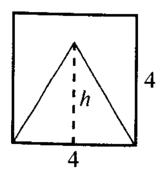
Quantity A	Quantity B
The probability of getting one head	The probability of getting two heads

EXAMPLE 32

Quantity A	Quantity B
The time it takes to drive 40 miles at 35 mph	The time it takes to drive 35 miles at 40 mph

Here's the important point to remember: don't choose D because *you* can't determine which quantity is bigger; choose D only if *nobody* could determine it. *You* may or may not know how to compute the number of seconds in a day, the area of an equilateral triangle, or a certain probability, but *these calculations can be made.*

SOLUTIONS 29–32

Direct Calculation	Solution Using Various TACTICS
29. Recall the facts you need and calculate. 60 seconds = 1 minute, 60 minutes = 1 hour, 24 hours = 1 day, 365 days = 1 year, and 100 years = 1 century. Quantity A: $60 \times 60 \times 24 = 86{,}400$ Quantity B: $365 \times 100 = 36{,}500$ Even if we throw in some days for leap years, the answer is clearly **A**.	29. The point of TACTIC 6 is that even if you have no idea how to calculate the number of seconds in a day, you can eliminate two choices. The answer *cannot* be D, and it would be an incredible coincidence if these two quantities were actually equal, so don't choose C. *Guess* between A and B.
30. Calculate both areas. (See KEY FACT J15 for the easy way to find the area of an equilateral triangle.) Quantity A: $A = s^2 = 4^2 = 16$ Quantity B: $A = \frac{s^2\sqrt{3}}{4} = \frac{4^2\sqrt{3}}{4} = 4\sqrt{3}$; and *twice A* is $8\sqrt{3}$. Since $\sqrt{3} \approx 1.7$, $8\sqrt{3} \approx 13.6$. The answer is **A**.	30. Use TACTIC 5: don't calculate—draw a diagram and then compare.  Since the height of the triangle is less than 4, its area is less than $\frac{1}{2}(4)(4) = 8$, and twice its area is less than 16, the area of the square. The answer is **A**. (If you don't see that, and just have to guess in order to move on, be sure not to guess D.)

Direct Calculation	Solution Using Various TACTICS
31. When a coin is flipped 3 times, there are 8 possible outcomes: HHH, HHT, HTH, HTT, THH, THT, TTH, and TTT. Of these, 3 have one head and 3 have two heads. Each probability is $\frac{3}{8}$. The answer is **C**.	31. Don't forget TACTIC 5. Even if you know how, you don't *have to* calculate the probabilities. When 3 coins are flipped, getting two heads means getting one tail. Therefore, the probability of two heads equals the probability of one tail, which by symmetry equals the probability of one head. The answer is **C**. (If you don't remember anything about probability, TACTIC 5 at least allows you to eliminate D before you guess.)
32. Since $d = rt$, $t = \frac{d}{r}$ [see Sect. 11-H]. Quantity A: $\frac{40}{35}$ hours—more than 1 hour. Quantity B: $\frac{35}{40}$ hours—less than 1 hour. The answer is **A**.	32. You *do* need to know these formulas, but *not* for this problem. At 35 mph it takes *more than an hour* to drive 40 miles. At 40 mph it takes *less than an hour* to drive 35 miles. Choose **A**.

Practice Exercises

Quantitative Comparison Questions

- Ⓐ Quantity A is greater.
- Ⓑ Quantity B is greater.
- Ⓒ Quantities A and B are equal.
- Ⓓ It is impossible to determine which quantity is greater.

	Quantity A	Quantity B
1.	197 + 398 + 586	203 + 405 + 607

$x > 0$

	Quantity A	Quantity B
2.	$10x$	$\frac{10}{x}$

	Quantity A	Quantity B
3.	The time that it takes to type 7 pages at a rate of 6 pages per hour	The time that it takes to type 6 pages at a rate of 7 pages per hour

$cd < 0$

	Quantity A	Quantity B
4.	$(c + d)^2$	$c^2 + d^2$

a, *b*, and *c* are the measures of the angles of isosceles triangle *ABC*.
x, *y*, and *z* are the measures of the angles of right triangle *XYZ*.

	Quantity A	Quantity B
5.	The average of *a*, *b*, and *c*	The average of *x*, *y*, and *z*

$b < 0$

	Quantity A	Quantity B
6.	$6b$	b^6

	Quantity A	Quantity B
7.	The area of a circle whose radius is 17	The area of a circle whose diameter is 35

Line *k* goes through (1,1) and (5,2).
Line *m* is perpendicular to *k*.

	Quantity A	Quantity B
8.	The slope of line *k*	The slope of line *m*

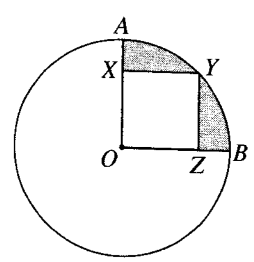
x is a positive integer

	Quantity A	Quantity B
9.	The number of multiples of 6 between 100 and $x + 100$	The number of multiples of 9 between 100 and $x + 100$

$x + y = 5$
$y - x = -5$

	Quantity A	Quantity B
10.	y	0

	Quantity A	Quantity B
11.	$\frac{7}{8}$	$\left(\frac{7}{8}\right)^5$



O is the center of the circle of radius 6. *OXYZ* is a square.

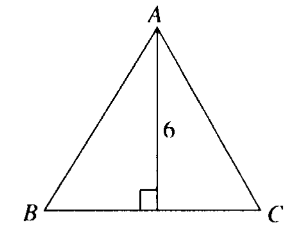
	Quantity A	Quantity B
12.	The area of the shaded region	12

The number of square inches in the surface area of a cube is equal to the number of cubic inches in its volume.

	Quantity A	Quantity B
13.	The length of an edge of the cube	6 inches

$1 < x < 4$

	Quantity A	Quantity B
14.	πx	x^2



$AB = AC$

	Quantity A	Quantity B
15.	The area of $\triangle ABC$	3

ANSWER KEY

1. **B**	4. **B**	7. **B**	10. **C**	13. **C**
2. **D**	5. **C**	8. **A**	11. **A**	14. **D**
3. **A**	6. **B**	9. **D**	12. **B**	15. **D**

ANSWER EXPLANATIONS

The direct mathematical solution to a problem is almost always the preferable one, so it is given first. It is often followed by one or more alternative solutions, indicated by a double asterisk (**), based on the various tactics discussed in this chapter. Occasionally, a solution based on one of the tactics is much easier than the straightforward one. In that case, it is given first.

1. **(B)** Using the onscreen calculator, this can easily be solved in 20 or 30 seconds by adding, but in only 5 seconds by thinking! Use TACTIC 5: don't calculate; compare. Each of the three numbers in Quantity B is greater than the corresponding numbers in Quantity A.

2. **(D)** Use TACTIC 1. When $x = 1$, the quantities are equal; when $x = 2$, they aren't.
   **Use TACTIC 3

	Quantity A	Quantity B
	$10x$	$\frac{10}{x}$
Multiply each quantity by x (this is OK since $x > 0$):	$10x^2$	10
Divide each quantity by 10:	x^2	1

This is a much easier comparison. x^2 *could* equal 1, but doesn't have to. The answer is Choice D.

3. **(A)** You can easily calculate each of the times — divide 7 by 6 to evaluate Quantity A, and 6 by 7 in Quantity B. However, it is easier to just observe that Quantity A is more than one hour, whereas Quantity B is less than one hour.

4. **(B)** Use TACTIC 3

	Quantity A	Quantity B
Expand Quantity A:	$(c + d)^2 =$ $c^2 + 2cd + d^2$	$c^2 + d^2$
Subtract $c^2 + d^2$ from each quantity:	$2cd$	0

Since it is given that $cd < 0$, so is $2cd$.

**If you can't expand $(c + d)^2$, then use TACTIC 1. Replace c and d with numbers satisfying $cd < 0$.

	Quantity A	Quantity B	Compare	Eliminate
Let $c = 1$ and $d = -1$	$(1 + -1)^2 = 0$	$1^2 + (-1)^2 = 1 + 1 = 2$	B is greater	A and C
Let $c = 3$ and $d = -5$	$(3 + -5)^2 = (-2)^2 = 4$	$3^2 + (-5)^2 = 9 + 25 = 34$	B is greater	

Both times Quantity B was greater: choose B.

5. **(C)** The average of 3 numbers is their sum divided by 3. Since in *any* triangle the sum of the measures of the 3 angles is 180°, the average in each quantity is equal to $180 \div 3 = 60$.

**Use TACTIC 1. Pick values for the measures of the angles. For example, in isosceles $\triangle ABC$ choose 70, 70, 40; in right $\triangle XYZ$, choose 30, 60, 90. Each average is 60. Choose C.

6. **(B)** Since $b < 0$, $6b$ is negative, whereas b^6 is positive.

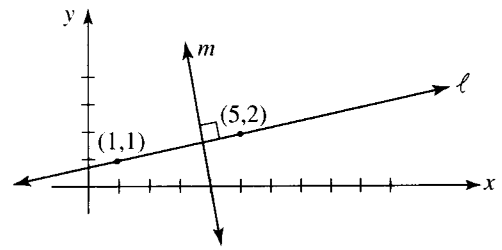
**Use TACTIC 1. Replace b with numbers satisfying $b < 0$.

	Quantity A	Quantity B	Compare	Eliminate
Let $b = -1$	$6(-1) = -6$	$(-1)^6 = 1$	B is greater	A and C
Let $b = -2$	$6(-2) = -12$	$(-2)^6 = 64$	B is greater	

Both times Quantity B was greater: choose B.

7. **(B)** Use TACTIC 5: don't calculate the two areas; compare them. The circle in Quantity A is the area of a circle whose radius is 17 and whose diameter is 34. Quantity B is the area of a circle whose diameter is 35, and so is clearly greater.

8. **(A)** Use TACTIC 5: don't calculate either slope. Quickly, make a rough sketch of line k, going through (1,1) and (5,2), and draw line m perpendicular to it.



Line k has a positive slope (it slopes upward), whereas line m has a negative slope (it slopes downward). Quantity A is greater.

[Note: The slope of k is $\frac{1}{4}$ and the slope of m is -4, but you don't need to calculate either one. See Section 11-N for all the facts you need to know about slopes.]

**If you don't know this fact about slopes, use TACTIC 6. The answer cannot be Choice D, and if two lines intersect, their slopes cannot be equal, so eliminate Choice C. Guess Choice A or B.

9. **(D)** Every sixth integer is a multiple of 6 and every ninth integer is a multiple of 9, so in a large interval there will be many more multiples of 6. But in a very small interval, there might be none or possibly just one of each.

**Use TACTIC 1. Let $x = 1$. Between 100 and 101 there are *no* multiples of 6 and *no* multiples of 9. Eliminate Choices A and B. Choose a large number for x: 100, for example. Between 100 and 200 there are many more multiples of 6 than there are multiples of 9. Eliminate Choice C.

10. **(C)** Add the equations.

$$\begin{array}{r} x + y = 5 \\ +\ y - x = -5 \\ \hline 2y = 0 \end{array}$$

Since $2y = 0$, $y = 0$.

**Use TACTIC 4. Could $y = 0$? In each equation, if $y = 0$, then $x = -5$. So, y can equal 0. Eliminate Choices A and B, and either guess between Choices C and D or continue. Must $y = 0$? Yes, when you have two linear equations in two variables, there is only one solution, so nothing else is possible.

11. **(A)** With a calculator, you can multiply $\frac{7}{8} \times \frac{7}{8} \times \frac{7}{8} \times \frac{7}{8} \times \frac{7}{8}$, but it is annoying and time-consuming. However, you can avoid the arithmetic, if you know KEY FACT A24:

If $0 < x < 1$ and $n > 1$, then $x^n < x$.

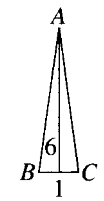
Since $\frac{7}{8} < 1$, then $\left(\frac{7}{8}\right)^5 < \frac{7}{8}$.

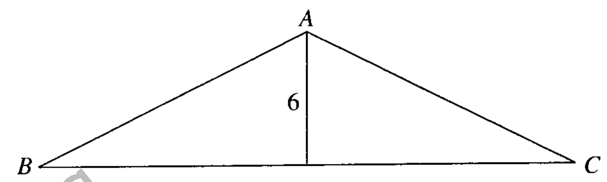
12. **(B)** The area of the shaded region is the area of quarter-circle AOB minus the area of the square. Since $r = OA = 6$, the area of the quarter-circle is $\frac{1}{4}\pi r^2 = \frac{1}{4}36\pi = 9\pi$. OY, the diagonal of the square, is 6 (since it is a radius of the circle), so OZ, the side of the square, is $\frac{6}{\sqrt{2}}$ [See KEY FACT J8]. So the area of the square is $\left(\frac{6}{\sqrt{2}}\right)^2 = \frac{36}{2} = 18$. Finally, the area of the shaded region is $9\pi - 18$, which is approximately 10.
**The solution above requires several steps. [See Sections 11-J, K, L to review any of the facts used.] If you can't reason through this, you still should be able to answer this question correctly. Use TACTIC 6. The shaded region has a definite area, which is either 12, more than 12, or less than 12. Eliminate D. Also, the area of a curved region almost always involves π, so assume the area isn't exactly 12. Eliminate Choice C. You can now *guess* between Choices A and B, but if you trust the diagram and know a little bit you can improve your guess. If you know that the area of the circle is 36π, so that the quarter-circle is 9π or about 28, you can estimate the shaded region. It's well less than half of the quarter-circle, so less than 14 and probably less than 12. Guess Choice B.

13. **(C)** Use TACTIC 4. Could the edge be 6? Test it. If each edge is 6, the area of each face is $6 \times 6 = 36$, and since a cube has 6 faces, the total surface area is $6 \times 36 = 216$. The volume is $6^3 = 216$. So the quantities could be equal. Eliminate Choices A and B. If you have a sense that this is the only cube with this property, choose C. In fact, if you had no idea how to do this, you might use TACTIC 6, assume that there is only one way, eliminate Choice D, and then guess C. The direct solution is simple enough if you know the formulas. If e is the length of an edge of the cube, then the area is $6e^2$ and the volume is e^3: $6e^2 = e^3 \Rightarrow 6 = e$.

14. **(D)** There are several ways to answer this question. Use TACTIC 1: plug in a number for x. If $x = 2$, Quantity A is 2π, which is slightly more than 6, and Quantity B is $2^2 = 4$. Quantity A is greater: eliminate Choices B and C. Must Quantity A be greater? If the only other number you try is $x = 3$, you'll think so, because $3^2 = 9$, but $3\pi > 9$. But remember, x does not have to be an integer: $3.9^2 > 15$, whereas $3.9\pi < 4\pi$, which is a little over 12.
**Use TACTIC 4. Could $\pi x = x^2$? Yes, if $x = \pi$. Must $x = \pi$? No.
**Use TACTIC 3. Divide each quantity by x: Now Quantity A is π and Quantity B is x. Which is bigger, π or x? We cannot tell.

15. **(D)** Use TACTIC 4. Could the area of $\triangle ABC = 3$? Since the height is 6, the area would be 3 only if the base were 1: $\frac{1}{2}(1)(6) = 3$. Could $BC = 1$? Sure (see the figure). Must the base be 1? Of course not.





Data Interpretation Questions

CHAPTER **10**

Three of the 20 questions in each quantitative section of the GRE are data interpretation questions. As their name suggests, these questions are always based on the information that is presented in some form of a graph or a chart. Occasionally, the data are presented in a chart or table, but much more often, they are presented graphically. The most common types of graphs are

- line graphs
- bar graphs
- circle graphs

In each section, the data interpretation questions are three consecutive questions, say questions 14, 15, and 16, all of which refer to the same set of graphs or charts.

When the first data interpretation question appears, either the graphs will be on the left-hand side of the screen, and the question will be on the right-hand side, or the graphs will be at the top of the screen and the question will be below them. It is possible, but unlikely, that you will have to scroll down in order to see all of the data. After you answer the first question, a second question will replace it on the right-hand side (or the bottom) of the screen; the graphs, of course, will still be on the screen for you to refer to.

The tactics discussed in this chapter can be applied to any type of data, no matter how they are displayed. In the practice exercises at the end of the chapter, there are data interpretation questions based on the types of graphs that normally appear on the GRE. Carefully, read through the answer explanations for each exercise, so that you learn the best way to handle each type of graph.

Infrequently, an easy data interpretation question will require only that you read the graph and find a numerical fact that is displayed. Usually, however, you will have to do some calculation on the data that you are analyzing. In harder questions, you may be given hypothetical situations and asked to make inferences based on the information provided in the given graphs.

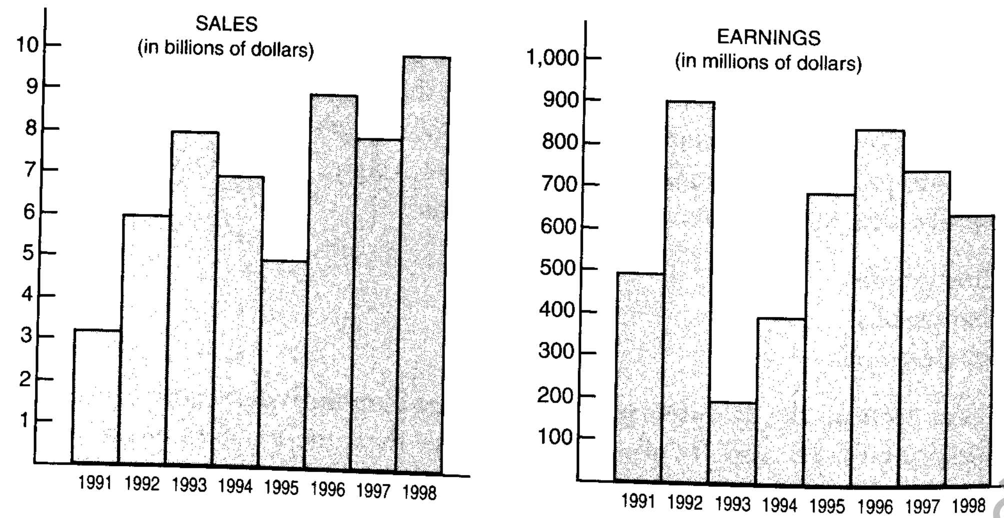
Most data interpretation questions are multiple-choice questions, but some could be multiple-answer or numeric entry questions. They are never quantitative comparisons.

Testing Tactics

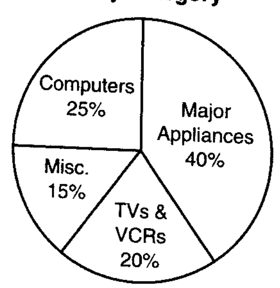
The four questions that follow will be used to illustrate the tactics that you should use in answering data interpretation questions. Remember, however, that on the GRE there will always be three questions that refer to a particular graph or set of graphs.

Questions 1–4 refer to the following graphs.

Sales and Earnings of XYZ Corporation 1991–1998



1998 Sales of XYZ Corporation by Category



1. What is the average (arithmetic mean) in billions of dollars of the sales of XYZ Corporation for the period 1991–1998?
Ⓐ 5.5 Ⓑ 6.0 Ⓒ 7.0 Ⓓ 8.0 Ⓔ 8.5

2. For which year was the percentage increase in earnings from the previous year the greatest?
Ⓐ 1992 Ⓑ 1993 Ⓒ 1994 Ⓓ 1995 Ⓔ 1996

3. Which of the following statements can be deduced from the data in the given charts and circle graph?

Indicate *all* such statements.

[A] Sales of major appliances in 1998 exceeded total sales in 1991.
[B] Earnings for the year in which earnings were greatest were more than sales for the year in which sales were lowest.
[C] If in 1998, the sales of major appliances had been 10% less, and the sales of computers had been 10% greater, the sales of major appliances would have been less than the sales of computers.

4. What was the ratio of earnings to sales in 1993?

$$\frac{\boxed{}}{\boxed{}}$$

TACTIC 1

First Read the Titles

When the first data interpretation question appears on the screen, do not even read it! Before you attempt to answer a data interpretation question, take 15 to 30 seconds to study the graphs. Try to get a general idea about the information that is being displayed.

Observe that the bar graphs on which questions 1–4 are based present two different sets of data. The bar graph on the left-hand side provides information about the sales of XYZ Corporation, and the right-hand graph provides information about the corporation's earnings. Also, note that whereas sales are given in billions of dollars, earnings are given in millions of dollars. Finally, the circle graph gives a breakdown by category of the sales of XYZ Corporation for one particular year.

TACTIC 2

Don't Confuse Percents and Numbers

Many students make mistakes on data interpretation questions because they don't distinguish between absolute numbers and percents. Although few students would look at the circle graph shown and think that XYZ Corporation sold 25 computers in 1998, many would mistakenly think that it sold 15% more major appliances than computers.

The problem is particularly serious when the questions involve percent increases or percent decreases. In question 2 you are not asked for the year in which the increase in earnings from the previous year was the greatest. You are asked for the year in which the percent increase in earnings was the greatest. A quick glance at the right-hand graph reveals that the greatest increase occurred from 1991 to 1992 when earnings jumped by $400 million. However, when we solve this problem in the discussion of TACTIC 3, you will see that Choice A is not the correct answer.

NOTE: Since many data interpretation questions involve percents, you should carefully study Section 11-C, and be sure that you know all of the tactics for solving percent problems. In particular, always try to use the number 100 or 1000, since it is so easy to mentally calculate percents of powers of 10.

TACTIC 3 Whenever Possible, Estimate

Although you have access to the onscreen calculator, when you take the GRE, you will not be expected to do complicated or lengthy calculations. Often, thinking and using some common sense can save you considerable time. For example, it may seem that in order to get the correct answer to question 2, you have to calculate five different percents. In fact, you only need to do one calculation, and that one you can do in your head!

Just looking at the Earnings bar graph, it is clear that the only possible answers are 1992, 1994, and 1995, the three years in which there was a significant increase in earnings from the year before. From 1993 to 1994 expenditures doubled, from \$200 million to \$400 million — an increase of 100%. From 1991 to 1992 expenditures increased by \$400 million (from \$500 million to \$900 million), but that is less than a 100% increase (we don't care how much less). From 1994 to 1995 expenditures increased by \$300 million (from \$400 million to \$700 million); but again, this is less than a 100% increase. The answer is **C**.

TACTIC 4 Do Each Calculation Separately

As in all multiple-answer questions, question 3 requires you to determine which of the statements are true. The key is to work with the statements individually.

To determine whether or not statement A is true, look at both the Sales bar graph and the circle graph. In 1998, total sales were \$10 billion, and sales of major appliances accounted for 40% of the total: 40% of \$10 billion = \$4 billion. This exceeds the \$3 billion total sales figure for 1991, so statement A is true.

In 1992, the year in which earnings were greatest, earnings were \$900 million. In 1991, the year in which sales were lowest, sales were \$3 billion, which is much greater than \$900 million. Statement B is false.

In 1998, sales of major appliances were \$4 billion. If they had been 10% less, they would have been \$3.6 billion. That year, sales of computers were \$2.5 billion (25% of \$10 billion). If computer sales had increased by 10%, sales would have increased by \$0.25 billion to \$2.75 billion. Statement C is false.

The answer is **A**.

TACTIC 5 Use Only the Information Given

You must base your answer to each question only on the information in the given charts and graphs. It is unlikely that you have any preconceived notion as to the sales of XYZ Corporation, but you might think that you know the population of the United States for a particular year or the percent of women currently in the workplace. If your knowledge contradicts any of the data presented in the graphs, ignore what you know. First of all, you may be mistaken; but more important, the data may refer to a different, unspecified location or year. In any event, *always* base your answers on the given data.

TACTIC 6 Always Use the Proper Units

In answering question 4, observe that earnings are given in millions, while sales are in billions. If you answer too quickly, you might say that in 1993 earnings were 200 and sales were 8, and conclude that the desired ratio is $\frac{200}{8} = \frac{25}{1}$. You will avoid this mistake if you keep track of units: earnings were 200 *million* dollars, whereas sales were 8 *billion* dollars. The correct ratio is

$$\frac{2\cancel{00,000,000}}{8\cancel{,000,000,000}} = \frac{2}{80} = \frac{1}{40}.$$

Enter 1 in the box for the numerator and 40 in the box for the denominator.

TACTIC 7 Be Sure That Your Answer Is Reasonable

Before clicking on your answer, take a second to be sure that it is reasonable. For example, in question 4, Choices D and E are unreasonable. From the logic of the situation, you should realize that earnings can't exceed sales. The desired ratio, therefore, must be less than 1. If you use the wrong units (see TACTIC 6, above), your initial thought would be to choose D. By testing your answer for reasonableness, you will realize that you made a mistake.

Remember that if you don't know how to solve a problem, you should always guess. Before guessing, however, check to see if one or more of the choices are unreasonable. If so, eliminate them. For example, if you forget how to calculate a percent increase, you would have to guess at question 2. But before guessing wildly, you should at least eliminate Choice B, since from 1992 to 1993 earnings decreased.

TACTIC 8 Try to Visualize the Answer

Because graphs and tables present data in a form that enables you to readily see relationships and to make quick comparisons, you can often avoid doing any calculations. Whenever possible, use your eye instead of your computational skills.

For example, to answer question 1, rather than reading the sales figures in the bar graph on the left for each of the eight years, adding them, and then dividing by 8, visualize the situation. Where could you draw a horizontal line across the graph so that there would be the same amount of gray area above the line as white area below it? Imagine a horizontal line drawn through the 7 on the vertical axis. The portions of the bars above the line for 1993 and 1996–1998 are just about exactly the same size as the white areas below the line for 1991, 1992, and 1994. The answer is **C**.

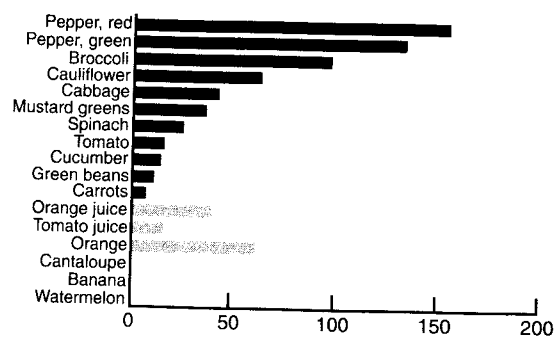
Practice Exercises

Data Interpretation Questions

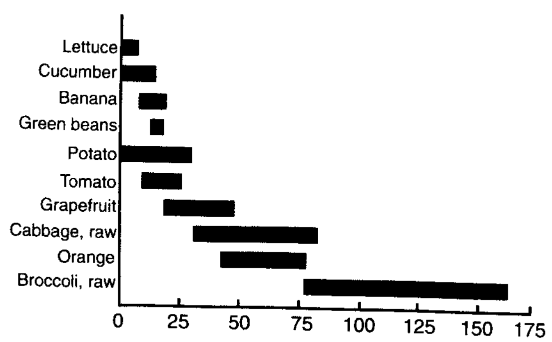
On the GRE there will typically be three questions based on any set of graphs. Accordingly, in each section of the model tests in this book, there are three data interpretation questions, each referring to the same set of graphs. However, to illustrate the variety of questions that can be asked, in this exercise set, for some of the graphs there are only two questions.

Questions 1–2 refer to the following graphs.

Vitamin C Content of Foods



Source: U.S. Department of Agriculture.



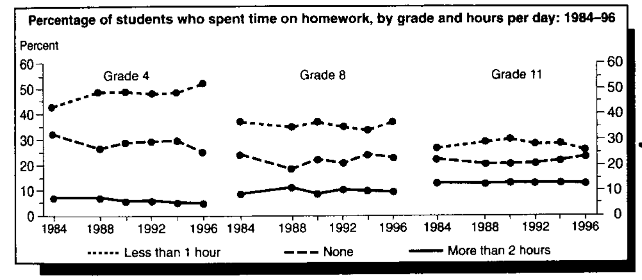
Source: U.S. Department of Agriculture.

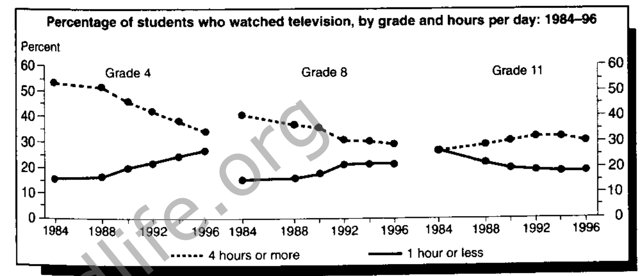
1. What is the ratio of the amount of Vitamin C in 500 grams of orange to the amount of Vitamin C in 500 grams of orange juice?
   Ⓐ 4:7
   Ⓑ 1:1
   Ⓒ 7:4
   Ⓓ 2:1
   Ⓔ 4:1

2. How many grams of tomato would you have to eat to be certain of getting more vitamin C than you would get by eating 100 grams of raw broccoli?
   Ⓐ 300
   Ⓑ 500
   Ⓒ 750
   Ⓓ 1200
   Ⓔ 1650

Questions 3–4 refer to the following graphs.

Percentage of students who reported spending time on homework and watching television





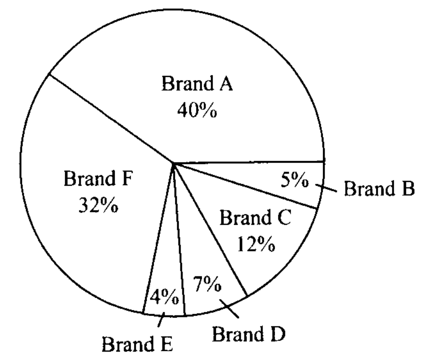
SOURCE: U.S. Department of Education.

3. In 1996, what percent of fourth-graders did between 1 and 2 hours of homework per day?
   Ⓐ 5%
   Ⓑ 15%
   Ⓒ 25%
   Ⓓ 40%
   Ⓔ 55%

4. If in 1984 there were 2,000,000 eleventh-graders, and if between 1984 and 1996 the number of eleventh-graders increased by 10%, then approximately how many more eleventh-graders watched 1 hour or less of television in 1996 than in 1984?
   Ⓐ 25,000
   Ⓑ 50,000
   Ⓒ 75,000
   Ⓓ 100,000
   Ⓔ 150,000

Questions 5–6 refer to the following graph.

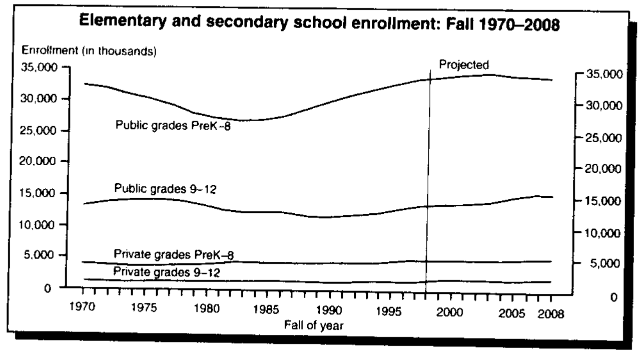
Total Sales of Coast Corporation in 2000: $1,000,000



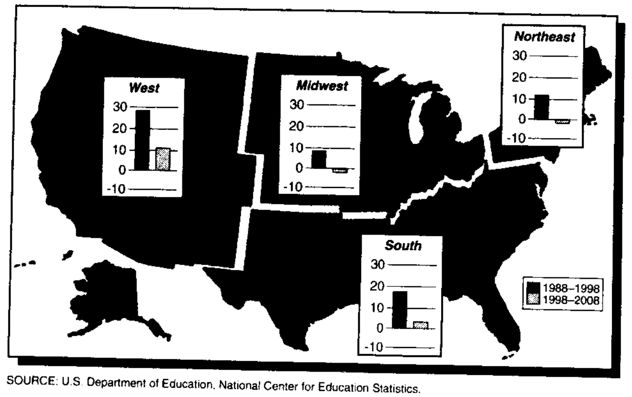
5. If the above circle graph were drawn to scale, then which of the following is closest to the difference in the degree measurements of the central angle of the sector representing Brand C and the central angle of the sector representing Brand D?
   Ⓐ 5°
   Ⓑ 12°
   Ⓒ 18°
   Ⓓ 25°
   Ⓔ 43°

6. The total sales of Coast Corporation in 2005 were 50% higher than in 2000. If the dollar value of the sales of Brand A was 25% higher in 2005 than in 2000, then the sales of Brand A accounted for what percentage of total sales in 2005?
   Ⓐ 20%
   Ⓑ 25%
   Ⓒ $33\frac{1}{3}$%
   Ⓓ 40%
   Ⓔ 50%

Questions 7–8 refer to the following graphs.



Projected percentage change in public elementary and secondary school enrollment, by region: Fall 1988 to 2008



SOURCE: U.S. Department of Education, National Center for Education Statistics.

7. To the nearest million, how many more students were enrolled in school — both public and private, preK–12 — in 1970 than in 1988?

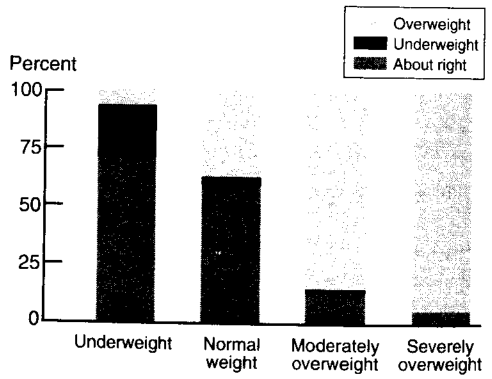
Ⓐ 3,000,000
Ⓑ 6,000,000
Ⓒ 10,000,000
Ⓓ 44,000,000
Ⓔ 51,000,000

8. In 1988 there were 40,000,000 public school students in the United States, of whom 22% lived in the West. Approximately, how many public school students are projected to be living in the West in 2008?

Ⓐ 9,000,000
Ⓑ 12,000,000
Ⓒ 15,000,000
Ⓓ 24,000,000
Ⓔ 66,000,000

Questions 9–10 refer to the following graph.

Perceptions of Body Weight Status



Actual weight status

Perceived compared with actual weight status of adult females.

Source: U.S. Department of Agriculture.

9. What percent of underweight adult females perceive themselves to be underweight?

[ ] %

10. The members of which of the four groups had the least accurate perception of their body weight?

Ⓐ Underweight
Ⓑ Normal weight
Ⓒ Moderately overweight
Ⓓ Severely overweight
Ⓔ It cannot be determined from the information given in the graph.

Questions 11–12 refer to the following table.

In 1979, residents of New York City paid both New York State and New York City tax. Residents of New York State who lived and worked outside of New York City paid only New York State tax.

Tax Rate Schedules for 1979

New York State: Taxable Income over	but not over	Amount of Tax					City of New York: Taxable Income over	but not over	Amount of Tax				
$ 0	$1,000			2%	of taxable income		$ 0	$1,000			0.9%	of taxable income	
1,000	3,000	$20	plus	3%	of excess over	$1,000	1,000	3,000	$ 9	plus	1.4%	of excess over	$1,000
3,000	5,000	80	plus	4%	of excess over	3,000	3,000	5,000	37	plus	1.8%	of excess over	3,000
5,000	7,000	160	plus	5%	of excess over	5,000	5,000	7,000	73	plus	2.0%	of excess over	5,000
7,000	9,000	260	plus	6%	of excess over	7,000	7,000	9,000	113	plus	2.3%	of excess over	7,000
9,000	11,000	380	plus	7%	of excess over	9,000	9,000	11,000	159	plus	2.5%	of excess over	9,000
11,000	13,000	520	plus	8%	of excess over	11,000	11,000	13,000	209	plus	2.7%	of excess over	11,000
13,000	15,000	680	plus	9%	of excess over	13,000	13,000	15,000	263	plus	2.9%	of excess over	13,000
15,000	17,000	860	plus	10%	of excess over	15,000	15,000	17,000	321	plus	3.1%	of excess over	15,000
17,000	19,000	1,060	plus	11%	of excess over	17,000	17,000	19,000	383	plus	3.3%	of excess over	17,000
19,000	21,000	1,280	plus	12%	of excess over	19,000	19,000	21,000	449	plus	3.5%	of excess over	19,000
21,000	23,000	1,520	plus	13%	of excess over	21,000	21,000	23,000	519	plus	3.8%	of excess over	21,000
23,000		1,780	plus	14%	of excess over	23,000	23,000	25,000	595	plus	4.0%	of excess over	23,000
							25,000		675	plus	4.3%	of excess over	25,000

11. In 1979 how much tax, in dollars, would a resident of New York State who lived and worked outside New York City have paid on a taxable income of $16,100?

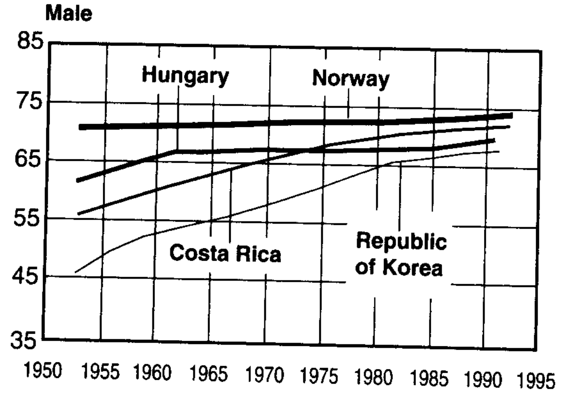
[ ] dollars

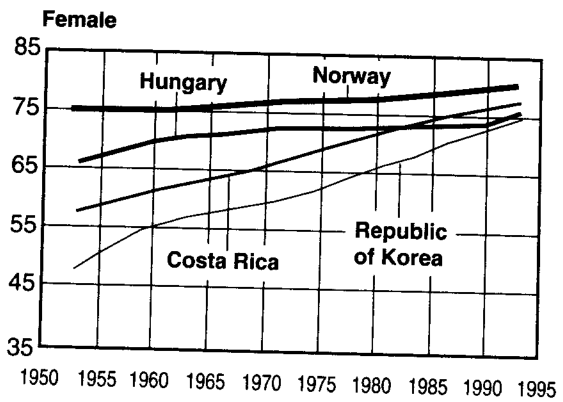
12. In 1979, how much more total tax would a resident of New York City who had a taxable income of $36,500 pay, compared to a resident of New York City who had a taxable income of $36,000?

Ⓐ $21.50
Ⓑ $43
Ⓒ $70
Ⓓ $91.50
Ⓔ $183

Questions 13–14 refer to the following tables.

Years of Life Expectancy at Birth
(Life expectancy in years)





Source: U.S. Bureau of the Census, Center for International Research.

13. For how many of the countries listed in the graphs is it true that the life expectancy of a female born in 1955 was higher than the life expectancy of a male born in 1990?

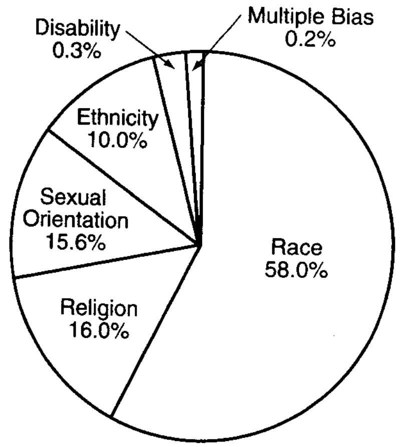
Ⓐ None
Ⓑ 1
Ⓒ 2
Ⓓ 3
Ⓔ 4

14. By sex and nationality, who had the greatest increase in life expectancy between 1955 and 1990?

Ⓐ A Korean female
Ⓑ A Korean male
Ⓒ A Costa Rican female
Ⓓ A Costa Rican male
Ⓔ A Norwegian female

Questions 15–16 refer to the following graph.

Bias-Motivated Offenses 1998
Percent Distribution



Source: U.S. Department of Justice, Federal Bureau of Investigation.

15. If in 1998 there were 10,000 bias-motivated offenses based on ethnicity, how many more offenses were based on religion than on sexual orientation?

Ⓐ 4
Ⓑ 40
Ⓒ 400
Ⓓ 4000
Ⓔ 40,000

16. If after further analysis it was determined that between 25% and 50% of the offenses included under Religion were, in fact, not bias-motivated, and those offenses were removed from the study, which of the following could be the percentage of bias-motivated offenses based on race?

Indicate *all* such percentages.

[A] 59%
[B] 60%
[C] 61%
[D] 62%
[E] 63%
[F] 64%
[G] 65%

ANSWER KEY

1. **C**	4. **E**	7. **B**	10. **A**	13. **B**	16. **C, D, E**
2. **E**	5. **C**	8. **B**	11. **970**	14. **A**	
3. **B**	6. **C**	9. **22**	12. **D**	15. **C**	

ANSWER EXPLANATIONS

1. **(C)** According to the graph on the left, there are approximately 70 milligrams of vitamin C in 100 grams of orange and 40 milligrams in the same amount of orange juice. This is a ratio of 70:40 = 7:4. Since the question refers to the same amount of orange and orange juice (500 grams), the ratio is unchanged.

2. **(E)** From the graph on the right, you can see that by eating 100 grams of raw broccoli, you could receive as much as 165 milligrams of vitamin C. Since 100 grams of tomato could have as little as 10 milligrams of vitamin C, you would have to eat 1650 grams of tomato to be sure of getting 165 milligrams of vitamin C.

3. **(B)** From the top graph, we see that among fourth-graders in 1996:
25% did no homework;
55% did less than 1 hour;
5% did more than 2 hours.
This accounts for 85% of the fourth-graders; the other 15% did between 1 and 2 hours of homework per day.

4. **(E)** In 1984, approximately 540,000 eleventh-graders watched television 1 hour or less per day (27% of 2,000,000). By 1996, the number of eleventh-graders had increased by 10% to 2,200,000, but the percent of them who watched television 1 hour or less per day decreased to about 18%: 18% of 2,200,000 is 396,000. This is a decrease of 144,000, or approximately 150,000.

5. **(C)** The central angle of the sector representing Brand C is 12% of 360°:
$(0.12) \times 360° = 43.2°$
The central angle of the sector representing Brand D is 7% of 360°.
$(0.7) \times 360° = 25.2°$
Finally, $43.2° \times 25.2° = 18°$
**Note this can be done in one step by noticing that the percentage difference between Brands C and D is 5% and 5% of 360 is $(0.05) \times 380 = 18$.

6. **(C)** Since total sales in 2000 were \$1,000,000, in 2005 sales were \$1,500,000 (a 50% increase).
In 2000, sales of Brand A were \$400,000 (40% of \$1,000,000).
In 2005 sales of Brand A were \$500,000 (25% or $\frac{1}{4}$ more than in 2000).
Finally, \$500,000 is $\frac{1}{3}$ or $33\frac{1}{3}$% of \$1,500,000.

7. **(B)** Reading from the top graph, we get the following enrollment figures:

	1970	1988
Public PreK–8	33,000,000	28,000,000
Public 9–12	13,000,000	12,000,000
Private PreK–8	4,000,000	4,000,000
Private 9–12	1,000,000	1,000,000
Total	51,000,000	45,000,000

51,000,000 – 45,000,000 = 6,000,000.

8. **(B)** In 1988, 8,800,000 (22% of 40,000,000) students lived in the West. From 1988–1998 this figure increased by 27% — for simplicity use 25%: an additional 2,200,000 students; so the total was then 11,000,000. The projected increase from 1998–2008 is about 10%, so the number will grow by 1,100,000 to 12,100,000.

9. **22** The bar representing underweight adult females who perceive themselves to be underweight extends from about 70% to about 95%, a range of approximately 25%. Choice B is closest.

10. **(A)** Almost all overweight females correctly considered themselves to be overweight; and more than half of all females of normal weight correctly considered themselves "about right." But nearly 70% of underweight adult females inaccurately considered themselves "about right."

11. **970** Referring only to the New York State table, we see that the amount of tax on a taxable income between $15,000 and $17,000 was $860 plus 10% of the excess over $15,000. Therefore, the tax on $16,100 is $860 plus 10% of $1,100 = $860 + $110 = $970.

12. **(D)** According to the tables, each additional dollar of taxable income over $25,000 was subject to a New York State tax of 14% and a New York City tax of 4.3%, for a total tax of 18.3%. Therefore, an additional $500 in taxable income would have incurred an additional tax of 0.183×500 = $91.50.

13. **(B)** In Norway, the life expectancy of a female born in 1955 was 75 years, which is greater than the life expectancy of a male born in 1990. In Hungary, the life expectancy of a female born in 1955 was 66 years, whereas the life expectancy of a male born in 1990 was greater than 67. In the other two countries, the life expectancy of a female born in 1955 was less than 65 years, and the life expectancy of a male born in 1990 was greater than 65.

14. **(A)** The life expectancy of a Korean female born in 1955 was about 51 and in 1990 it was about 74, an increase of 23 years. This is greater than any other nationality and sex.

15. **(C)** Since there were 10,000 bias-motivated offenses based on ethnicity, and that represents 10% of the total, there were 100,000 bias-motivated offenses in total. Of these, 16,000 (16% of 100,000) were based on religion, and 15,600 (15.6% of 100,000) were based on sexual orientation. The difference is 400.

16. **(C)**, **(D)**, **(E)** Since this is a question about percentages, assume that the total number of bias-motivated offenses in 1998 was 100, of which 16 were based on religion and 58 were based on race.

- If 8 of the religion-based offenses (50% of 16) were deleted, then there would have been 92 offenses in all, of which 58 were based on race.

  $\frac{58}{92} = 0.6304 = 63.04\%$

- If 4 of the religion-based offenses (25% of 16) were deleted, then there would have been 96 offenses in all, of which 58 were based on race.

  $\frac{58}{96} = 0.6041 = 60.41\%$

Only choices C, D, and E lie between 60.41% and 63.04%.

Mathematics Review

CHAPTER 11

The mathematics questions on the GRE General Test require a working knowledge of mathematical principles, including an understanding of the fundamentals of algebra, plane geometry, and arithmetic, as well as the ability to translate problems into formulas and to interpret graphs. Very few questions require any math beyond what is typically taught in the first two years of high school, and even much of that is not tested. The following review covers those areas that you definitely need to know.

This chapter is divided into 15 sections, labeled 11-A through 11-O. For each question on the Diagnostic Test and the two Model Tests, the Answer Key indicates which section of Chapter 11 you should consult if you need help on a particular topic.

How much time you initially devote to reviewing mathematics should depend on your math skills. If you have always been a good math student and you have taken some math in college and remember most of your high school math, you can skip the instructional parts of this chapter for now. If while doing the Model Tests in Part 5 or on the accompanying CD-ROM, you find that you keep making mistakes on certain types of problems (averages, percents, circles, solid geometry, word problems, for example), or they take you too long, you should then study the appropriate sections here. Even if your math skills are excellent, and you don't need the review, you should complete the sample questions in those sections; they are an excellent source of additional GRE questions. If you know that your math skills are not very good and you have not done much math since high school, then it is advisable to review all of this material, including working out the problems, *before* tackling the model tests.

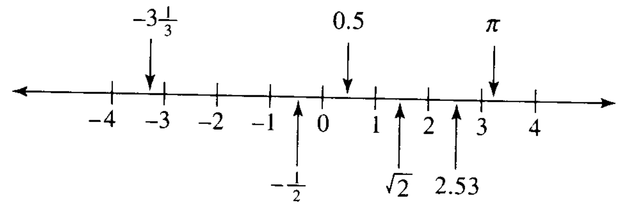
No matter how good you are in math, *you should carefully read and do the problems* in Chapters 7, 8, 9, and 10. For many of these problems, two solutions are given: the most direct mathematical solution and a second solution using one or more of the special tactics taught in these chapters.

Arithmetic

To do well on the GRE, you need to feel comfortable with most topics of basic arithmetic. In the first five sections of this chapter, we will review the basic arithmetic operations, signed numbers, fractions, decimals, ratios, percents, and averages. Since the GRE uses these concepts to test your reasoning skills, not your ability to perform tedious calculations, we will concentrate on the concepts and not on arithmetic drill. The solutions to more than one-third of the mathematics questions on the GRE ... Be sure to review them all.

11-A. BASIC ARITHMETIC CONCEPTS

Let's start by reviewing the most important sets of numbers and their properties. On the GRE the word *number* always means *real number*, a number that can be represented by a point on the number line.



Signed Numbers

The numbers to the right of 0 on the number line are called ***positive*** and those to the left of 0 are called ***negative***. Negative numbers must be written with a *negative sign* (−2); positive numbers can be written with a *plus sign* (+2) but are usually written without a sign (2). All numbers can be called ***signed numbers***.

KEY FACT A1

For any number *a*, exactly one of the following is true:

- ***a* is negative**
- ***a* = 0**
- ***a* is positive**

TIP

The absolute value of a number if *never* negative.

The ***absolute value*** of a number *a*, denoted $|a|$, is the distance between *a* and 0 on the number line. Since 3 is 3 units to the right of 0 on the number line and −3 is 3 units to the left of 0, both have an absolute value of 3:

- $|3| = 3$
- $|-3| = 3$

Two unequal numbers that have the same absolute value are called ***opposites***. So, 3 is the opposite of −3 and −3 is the opposite of 3.

KEY FACT A2

The only number that is equal to its opposite is 0.

EXAMPLE 1

$a - b = -(a - b)$

Quantity A	Quantity B
a	*b*

SOLUTION.
Since $-(a - b)$ is the opposite of $a - b$, $a - b = 0$, and so $a = b$. The answer is **C**.

In arithmetic we are basically concerned with the addition, subtraction, multiplication, and division of numbers. The third column of the following table gives the terms for the results of these operations.

Operation	Symbol	Result	Example
Addition	+	***Sum***	16 is the sum of 12 and 4 16 = 12 + 4
Subtraction	−	***Difference***	8 is the difference of 12 and 4 8 = 12 − 4
Multiplication*	×	***Product***	48 is the product of 12 and 4 48 = 12 × 4
Division	÷	***Quotient***	3 is the quotient of 12 and 4 3 = 12 ÷ 4

*Multiplication can be indicated also by a dot, parentheses, or the juxtaposition of symbols without any sign: $2^2 \cdot 2^4$, 3(4), 3(*x* + 2), 3*a*, 4*abc*.

Given any two numbers *a* and *b*, we can *always* find their sum, difference, product, and quotient, except that we may *never divide by zero*.

- 0 ÷ 7 = 0
- 7 ÷ 0 is meaningless

EXAMPLE 2

What is the sum of the product and quotient of 8 and 8?

Ⓐ 16 Ⓑ 17 Ⓒ 63 Ⓓ 64 Ⓔ 65

SOLUTION.
Product: 8 × 8 = 64. Quotient: 8 ÷ 8 = 1. Sum: 64 + 1 = **65** (**E**).

KEY FACT A3

- **The product of 0 and any number is 0. For any number *a*: *a* × 0 = 0.**
- **Conversely, if the product of two numbers is 0, *at least one* of them must be 0:**

$$ab = 0 \Rightarrow a = 0 \text{ or } b = 0.$$

EXAMPLE 3

Quantity A	Quantity B
The product of the integers from −7 to 2	The product of the integers from −2 to 7

SOLUTION.
Do not multiply. Each quantity is the product of 10 numbers, one of which is 0. So, by KEY FACT A3, each product is 0. The quantities are equal (**C**).

KEY FACT A4

The product and quotient of two positive numbers or two negative numbers are positive; the product and quotient of a positive number and a negative number are negative.

×	+	−
+	+	−
−	−	+

÷	+	−
+	+	−
−	−	+

$6 \times 3 = 18$ $\quad 6 \times (-3) = -18 \quad (-6) \times 3 = -18 \quad (-6) \times (-3) = 18$

$6 \div 3 = 2 \quad 6 \div (-3) = -2 \quad (-6) \div 3 = -2 \quad (-6) \div (-3) = 2$

To determine whether a product of more than two numbers is positive or negative, count the number of negative factors.

KEY FACT A5

- **The product of an *even* number of negative factors is positive.**
- **The product of an *odd* number of negative factors is negative.**

EXAMPLE 4

Quantity A	Quantity B
(−1)(2)(−3)(4)(−5)	(1)(−2)(3)(−4)(5)

SOLUTION.
Don't waste time multiplying. Quantity A is negative since it has 3 negative factors, whereas Quantity B is positive since it has 2 negative factors. The answer is **B**.

KEY FACT A6

- **The *reciprocal* of any nonzero number a is $\frac{1}{a}$.**
- **The product of any number and its reciprocal is 1:**

$$a \times \left(\frac{1}{a}\right) = 1.$$

KEY FACT A7

- **The sum of two positive numbers is positive.**
- **The sum of two negative numbers is negative.**
- **To find the sum of a positive and a negative number, find the difference of their absolute values and use the sign of the number with the larger absolute value.**

$$6 + 2 = 8 \qquad (-6) + (-2) = -8$$

To calculate either $6 + (-2)$ or $(-6) + 2$, take the *difference*, $6 - 2 = 4$, and use the sign of the number whose absolute value is 6. So,

$$6 + (-2) = 4 \qquad (-6) + 2 = -4$$

KEY FACT A8

The sum of any number and its opposite is 0:

$$a + (-a) = 0.$$

Many of the properties of arithmetic depend on the relationship between subtraction and addition and between division and multiplication.

KEY FACT A9

- **Subtracting a number is the same as adding its opposite.**
- **Dividing by a number is the same as multiplying by its reciprocal.**

$$a - b = a + (-b) \qquad a \div b = a \times \left(\frac{1}{b}\right)$$

Many problems involving subtraction and division can be simplified by changing them to addition and multiplication problems, respectively.

KEY FACT A10

To subtract signed numbers, change the problem to an addition problem, by changing the sign of what is being subtracted, and use KEY FACT A7.

$$2 - 6 = 2 + (-6) = -4 \qquad 2 - (-6) = 2 + (6) = 8$$

$$(-2) - (-6) = (-2) + (6) = 4 \qquad (-2) - 6 = (-2) + (-6) = -8$$

In each case, the minus sign was changed to a plus sign, and either the 6 was changed to −6 or the −6 was changed to 6.

Integers

The *integers* are $\{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$.
The *positive integers* are $\{1, 2, 3, 4, 5, \ldots\}$.
The *negative integers* are $\{\ldots, -5, -4, -3, -2, -1\}$.

TIP

0 is neither positive nor negative, but it *is* an integer.

There are five integers whose absolute value is less than 3—two negative integers (−2 and −1), two positive integers (1 and 2), and 0.

Consecutive integers are two or more integers written in sequence in which each integer is 1 more than the preceding integer. For example:

$$22, 23 \quad 6, 7, 8, 9 \quad -2, -1, 0, 1 \quad n, n+1, n+2, n+3$$

EXAMPLE 5

If the sum of three consecutive integers is less than 75, what is the greatest possible value of the smallest one?

Ⓐ 23 Ⓑ 24 Ⓒ 25 Ⓓ 26 Ⓔ 27

SOLUTION.
Let the numbers be n, $n + 1$, and $n + 2$. Then,

$$n + (n + 1) + (n + 2) = 3n + 3 \Rightarrow 3n + 3 < 75 \Rightarrow 3n < 72 \Rightarrow n < 24.$$

So, the most n can be is **23 (A)**.

CAUTION

Never assume that *number* means *integer*: 3 is not the only number between 2 and 4; there are infinitely many, including 2.5, 3.99, $\frac{10}{3}$, π, and $\sqrt{10}$.

EXAMPLE 6

If $2 < x < 4$ and $3 < y < 7$, what is the largest integer value of $x + y$?

SOLUTION.
If x and y are integers, the largest value is $3 + 6 = 9$. However, although $x + y$ is to be an integer, neither x nor y must be. If $x = 3.8$ and $y = 6.2$, then $x + y =$ **10**.

The sum, difference, and product of two integers are *always* integers; the quotient of two integers may be an integer, but it is not necessarily one. The quotient $23 \div 10$ can be expressed as $\frac{23}{10}$ or $2\frac{3}{10}$ or 2.3. If the quotient is to be an integer, we can say that the quotient is 2 and there is a ***remainder*** of 3. It depends upon our point of view. For example, if 23 dollars is to be divided among 10 people, each one will get \$2.30 (2.3 dollars); but if 23 books are to be divided among 10 people, each one will get 2 books and there will be 3 books left over (the remainder).

KEY FACT A11

If m and n are positive integers and if r is the remainder when n is divided by m, then n is r more than a multiple of m. That is, $n = mq + r$ where q is an integer and $0 \le r < m$.

EXAMPLE 7

How many positive integers less than 100 have a remainder of 3 when divided by 7?

SOLUTION.
To leave a remainder of 3 when divided by 7, an integer must be 3 more than a multiple of 7. For example, when 73 is divided by 7, the quotient is 10 and the remainder is 3: $73 = 10 \times 7 + 3$. So, just take the multiples of 7 and add 3. (*Don't forget that 0 is a multiple of 7.*)

$$\underline{0} \times 7 + 3 = 3; \quad \underline{1} \times 7 + 3 = 10;$$
$$\underline{2} \times 7 + 3 = 17; \quad \ldots;$$
$$\underline{13} \times 7 + 3 = 94$$

A total of **14** numbers.

Calculator Shortcut

The standard way to find quotients and remainders is to use long division; but on the GRE you *never* do long division: you use the onscreen calculator. To find the remainder when 100 is divided by 7, divide on your calculator: $100 \div 7 = 14.285714\ldots$. This tells you that the quotient is 14. (Ignore everything to the right of the decimal point.) To find the remainder, multiply $14 \times 7 = 98$, and then subtract: $100 - 98 = 2$.

EXAMPLE 8

If today is Saturday, what day will it be in 500 days?

Ⓐ Friday Ⓑ Saturday Ⓒ Sunday Ⓓ Monday Ⓔ Tuesday

SOLUTION.
The days of the week form a repeating sequence. Seven days (1 week), 70 days (10 weeks), 700 days (100 weeks) from Saturday it is again Saturday. If 500 were a multiple of 7, then the answer would be Choice B, Saturday. Is it? With your calculator divide 500 by 7: $500 \div 7 = 71.428\ldots$ So, 500 is not a multiple of 7; since $71 \times 7 = 497$, The quotient when 500 is divided by 7 is 71, and the remainder is 3. Therefore, 500 days is 3 days more than 71 complete weeks. 497 days from Saturday it will again be Saturday; three days later it will be **Tuesday, (E)**.

If a and b are integers, the following four terms are synonymous:

a is a ***divisor*** of b	a is a ***factor*** of b
b is ***divisible*** by a	b is a ***multiple*** of a

They all mean that when b is divided by a there is no remainder (or, more precisely, the remainder is 0). For example:

3 is a divisor of 12	3 is a factor of 12
12 is divisible by 3	12 is a multiple of 3

KEY FACT A12

Every integer has a finite set of factors (or divisors) and an infinite set of multiples.

The factors of 12: $-12, -6, -4, -3, -2, -1, 1, 2, 3, 4, 6, 12$
The multiples of 12: $\ldots, -48, -36, -24, -12, 0, 12, 24, 36, 48, \ldots$

The only positive divisor of 1 is 1. All other positive integers have at least 2 positive divisors: 1 and itself, and possibly many more. For example, 6 is divisible by 1 and 6, as well as 2 and 3, whereas 7 is divisible only by 1 and 7. Positive integers, such as 7, that have exactly 2 positive divisors are called ***prime numbers*** or ***primes***. The first ten primes are

2, 3, 5, 7, 11, 13, 17, 19, 23, 29

TIP

1 is *not* a prime.

Memorize this list — it will come in handy.

Positive integers greater than 1 that are not prime are called ***composite numbers***. It follows from the definition that every composite number has at least three distinct positive divisors. The first ten composite numbers are

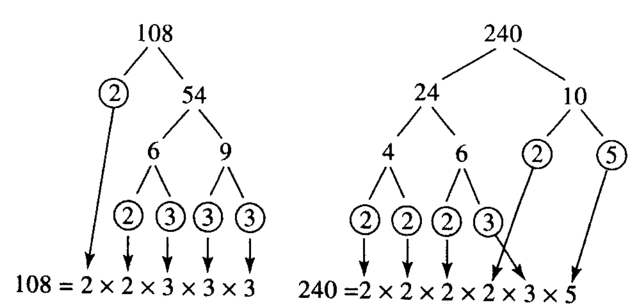
4, 6, 8, 9, 10, 12, 14, 15, 16, 18

KEY FACT A13

Every integer greater than 1 that is not a prime (i.e., every composite number) can be written as a product of primes.

To find the prime factorization of any integer, find any two factors; if they're both primes, you are done; if not, factor them. Continue until each factor has been written in terms of primes. A useful method is to make a *factor tree.*

For example, here are the prime factorizations of 108 and 240:



EXAMPLE 9

For any positive integer a, let $\lceil a \rfloor$ denote the smallest prime factor of a. Which of the following is equal to $\lceil 35 \rfloor$?

Ⓐ $\lceil 10 \rfloor$ Ⓑ $\lceil 15 \rfloor$ Ⓒ $\lceil 45 \rfloor$ Ⓓ $\lceil 55 \rfloor$ Ⓔ $\lceil 75 \rfloor$

SOLUTION.

Check the first few primes; 35 is not divisible by 2 or 3, but is divisible by 5, so 5 is the *smallest* prime factor of 35: $\lceil 35 \rfloor = 5$. Now check the five choices: $\lceil 10 \rfloor = 2$, and $\lceil 15 \rfloor$, $\lceil 45 \rfloor$, and $\lceil 75 \rfloor$ are all equal to 3. Only $\lceil \mathbf{55} \rfloor = 5$. The answer is **D.**

The ***least common multiple*** (**LCM**) of two or more integers is the smallest positive integer that is a multiple of each of them. For example, the LCM of 6 and 10 is 30. Infinitely many positive integers are multiples of both 6 and 10, including 60, 90, 180, 600, 6000, and 66,000,000, but 30 is the smallest one. The ***greatest common factor*** (**GCF**) or ***greatest common divisor*** (**GCD**) of two or more integers is the largest integer that is a factor of each of them. For example, the only positive integers that are factors of both 6 and 10 are 1 and 2, so the GCF of 6 and 10 is 2. For small numbers, you can often find their GCF and LCM by inspection. For larger numbers, KEY FACT A14 is very useful.

KEY FACT A14

The product of the GCF and LCM of two numbers is equal to the product of the two numbers.

TIP

It is usually easier to find the GCF than the LCM. For example, you might see immediately that the GCF of 36 and 48 is 12. You could then use KEY FACT A14 to find the LCM: since GCF × LCM = 36 × 48, then

$$\text{LCM} = \frac{\overset{3}{\cancel{36}} \times 48}{\underset{1}{\cancel{12}}} = 3 \times 48 = 144.$$

An easy way to find the GCF or LCM of two or more integers is to first get their prime factorizations.

- The GCF is the product of all the primes that appear in each factorization, using each prime the smallest number of times it appears in any of the factorizations.
- The LCM is the product of all the primes that appear in any of the factorizations, using each prime the largest number of times it appears in any of the factorizations.

For example, let's find the GCF and LCM of 108 and 240. As we saw:

$$108 = 2 \times 2 \times 3 \times 3 \times 3 \text{ and } 240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5.$$

- **GCF.** The primes that appear in both factorizations are 2 and 3: 2 appears twice in the factorization of 108 and 4 times in the factorization of 240, so we take it twice; 3 appears 3 times in the factorization of 108, but only once in the factorization of 240, so we take it just once. The GCF = $2 \times 2 \times 3 =$ **12**.
- **LCM.** Take one of the factorizations and add to it any primes from the other that are not yet listed. So, start with $2 \times 2 \times 3 \times 3 \times 3$ (108) and look at the primes from 240: there are four 2s; we already wrote two 2s, so we need two more; there is a 3 but we already have that; there is a 5, which we need. So, the LCM = $(2 \times 2 \times 3 \times 3 \times 3) \times (2 \times 2 \times 5) = 108 \times 20 =$ **2,160**.

EXAMPLE 10

What is the smallest number that is divisible by both 34 and 35?

[ ]

SOLUTION. We are being asked for the LCM of 34 and 35. By KEY FACT A14, LCM = $\frac{34 \times 35}{\text{GCF}}$. But the GCF is 1 since no number greater than 1 divides evenly into both 34 and 35. So, the LCM is 34 × 35 = **1,190**.

The ***even numbers*** are all the multiples of 2:

{..., –4, –2, 0, 2, 4, 6, ...}

The ***odd numbers*** are the integers not divisible by 2:

{..., –5, –3, –1, 1, 3, 5, ...}

TIP

The terms odd and even apply *only* to integers.

NOTE:

- Every integer (positive, negative, or 0) is either odd or even.
- 0 is an even integer; it is a multiple of 2. (0 = 0 × 2)
- 0 is a multiple of *every* integer. (0 = 0 × *n*)
- 2 is the only even prime number.

KEY FACT A15

The tables below summarize three important facts:

1. **If two integers are both even or both odd, their sum and difference are even.**
2. **If one integer is even and the other odd, their sum and difference are odd.**
3. **The product of two integers is even unless both of them are odd.**

+ and –	even	odd
even	even	odd
odd	odd	even

×	even	odd
even	even	even
odd	even	odd

Exponents and Roots

Repeated addition of the same number is indicated by multiplication:

$$17 + 17 + 17 + 17 + 17 + 17 + 17 = 7 \times 17$$

Repeated multiplication of the same number is indicated by an exponent:

$$17 \times 17 \times 17 \times 17 \times 17 \times 17 \times 17 = 17^7$$

In the expression 17^7, 17 is called the ***base*** and 7 is the ***exponent***.

At some time, you may have seen expressions such as 2^{-4}, $2^{\frac{1}{2}}$, or even $2^{\sqrt{2}}$. On the GRE, although the base, *b*, can be any number, the exponents you will see will almost always be positive integers.

KEY FACT A16

For any number *b*: $b^1 = b$, and $b^n = b \times b \times \cdots \times b$, where *b* is used as a factor *n* times.

(i) $2^5 \times 2^3 = (2 \times 2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2) = 2^8 = \mathbf{2^{5+3}}$

(ii) $\frac{2^5}{2^3} = \frac{2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2} = 2 \times 2 = 2^2 = \mathbf{2^{5-3}}$

(iii) $(2^2)^3 = (2 \times 2)^3 = (2 \times 2) \times (2 \times 2) \times (2 \times 2) = 2^6 = \mathbf{2^{2\times 3}}$

(iv) $2^3 \times 7^3 = (2 \times 2 \times 2) \times (7 \times 7 \times 7) = (2 \times 7)(2 \times 7)(2 \times 7) = \mathbf{(2 \times 7)^3}$

These four examples illustrate the following important ***laws of exponents*** given in KEY FACT A17.

KEY FACT A17

For any numbers *b* and *c* and positive integers *m* and *n*:

(i) $b^m b^n = b^{m+n}$ **(ii)** $\frac{b^m}{b^n} = b^{m-n}$ **(iii)** $(b^m)^n = b^{mn}$ **(iv)** $b^m c^m = (bc)^m$

TIP

Memorize the laws of exponents. They come up often on the GRE.

CAUTION

In (i) and (ii) the bases are the same and in (iv) the exponents are the same. None of these rules applies to expressions such as $7^5 \times 5^7$, in which both the bases and the exponents are different.

EXAMPLE 11

If $2^x = 32$, what is x^2?

Ⓐ 5 Ⓑ 10 Ⓒ 25 Ⓓ 100 Ⓔ 1024

SOLUTION.
To solve $2^x = 32$, just count (and keep track of) how many 2s you need to multiply to get 32: $2 \times 2 \times 2 \times 2 \times 2 = 32$, so $x = 5$ and $x^2 = $ **25 (C)**.

EXAMPLE 12

If $3^a \times 3^b = 3^{100}$, what is the average (arithmetic mean) of *a* and *b*?

[ ]

SOLUTION.

Since $3^a \times 3^b = 3^{a+b}$, we see that $a + b = 100 \Rightarrow \frac{a+b}{2} = \mathbf{50}$.

The next KEY FACT is an immediate consequence of KEY FACTS A4 and A5.

KEY FACT A18

For any positive integer n:

- $0^n = 0$
- **if a is positive, then a^n is positive**
- **if a is negative and n is even, then a^n is positive**
- **if a is negative and n is odd, then a^n is negative.**

EXAMPLE 13

Quantity A	Quantity B
$(-13)^{10}$	$(-13)^{25}$

SOLUTION.
Quantity A is positive and Quantity B is negative. So Quantity **A** is greater.

Squares and Square Roots

The exponent that appears most often on the GRE is 2. It is used to form the square of a number, as in πr^2 (the area of a circle), $a^2 + b^2 = c^2$ (the Pythagorean theorem), or $x^2 - y^2$ (the difference of two squares). Therefore, it is helpful to recognize the ***perfect squares***, numbers that are the squares of integers. The squares of the integers from 0 to 15 are as follows:

x	0	1	2	3	4	5	6	7
x^2	0	1	4	9	16	25	36	49

x	8	9	10	11	12	13	14	15
x^2	64	81	100	121	144	169	196	225

There are two numbers that satisfy the equation $x^2 = 9$: $x = 3$ and $x = -3$. The positive one, 3, is called the (***principal***) ***square root*** of 9 and is denoted by the symbol $\sqrt{9}$. Clearly, each perfect square has a square root: $\sqrt{0} = 0$, $\sqrt{36} = 6$, $\sqrt{81} = 9$, and $\sqrt{144} = 12$. But, it is an important fact that *every* positive number has a square root.

KEY FACT A19

For any positive number a, there is a positive number b that satisfies the equation $b^2 = a$. That number is called the square root of a and we write $b = \sqrt{a}$.

So, for any positive number a: $(\sqrt{a})^2 = \sqrt{a} \times \sqrt{a} = a$.

The only difference between $\sqrt{9}$ and $\sqrt{10}$ is that the first square root is an integer, while the second one isn't. Since 10 is a little more than 9, we should expect that $\sqrt{10}$ is a little more than $\sqrt{9} = 3$. In fact, $(3.1)^2 = 9.61$, which is close to 10, and $(3.16)^2 = 9.9856$, which is very close to 10. So, $\sqrt{10} \approx 3.16$. On the GRE you will *never* have to evaluate such a square root; if the solution to a problem involves a square root, that square root will be among the answer choices.

EXAMPLE 14

What is the circumference of a circle whose area is 10π?

Ⓐ 5π Ⓑ 10π Ⓒ $\pi\sqrt{10}$ Ⓓ $2\pi\sqrt{10}$ Ⓔ $\pi\sqrt{20}$

SOLUTION.
Since the area of a circle is given by the formula $A = \pi r^2$, we have

$$\pi r^2 = 10\pi \Rightarrow r^2 = 10 \Rightarrow r = \sqrt{10}.$$

The circumference is given by the formula $C = 2\pi r$, so $C = \mathbf{2\pi\sqrt{10}}$ **(D)**.

KEY FACT A20

For any positive numbers a and b:

- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

CAUTION

$\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$. For example:

$$5 = \sqrt{25} = \sqrt{9+16} \neq \sqrt{9} + \sqrt{16} = 3 + 4 = 7.$$

CAUTION

Although it is always true that $(\sqrt{a})^2 = a$, $\sqrt{a^2} = a$ is true *only if a* is positive:

$$\sqrt{(-5)^2} = \sqrt{25} = 5, \textit{ not } -5.$$

EXAMPLE 15

Quantity A	Quantity B
$\sqrt{x^{20}}$	$(x^5)^2$

SOLUTION.
Quantity A: Since $x^{10}x^{10} = x^{20}$, $\sqrt{x^{20}} = x^{10}$. Quantity B: $(x^5)^2 = x^{10}$. The quantities are equal **(C)**.

PEMDAS

When a calculation requires performing more than one operation, it is important to carry them out in the correct order. For decades students have memorized the sentence "Please Excuse My Dear Aunt Sally," or just the first letters, PEMDAS, to remember the proper order of operations. The letters stand for:

- Parentheses: first do whatever appears in parentheses, following PEMDAS within the parentheses if necessary.
- Exponents: next evaluate all terms with exponents.
- Multiplication and Division: then do all multiplications and divisions *in order from left to right* — *do not* multiply first and then divide.
- Addition and Subtraction: finally, do all additions and subtractions *in order from left to right* — do not add first and then subtract.

Here are some worked-out examples.

1. $12 + 3 \times 2 = 12 + 6 = 18$ [Multiply before you add.]
   $(12 + 3) \times 2 = 15 \times 2 = 30$ [First add in the parentheses.]
2. $12 \div 3 \times 2 = 4 \times 2 = 8$ [Just go from left to right.]
   $12 \div (3 \times 2) = 12 \div 6 = 2$ [First multiply inside the parentheses.]
3. $5 \times 2^3 = 5 \times 8 = 40$ [Do exponents first.]
   $(5 \times 2)^3 = 10^3 = 1000$ [First multiply inside the parentheses.]
4. $4 + 4 \div (2 + 6) = 4 + 4 \div 8 = 4 + .5 = 4.5$
   [First add in the parentheses, then divide, and finally add.]
5. $100 - 2^2(3 + 4 \times 5) = 100 - 2^2(23) = 100 - 4(23) = 100 - 92 = 8$
   [First evaluate what's inside the parentheses (using PEMDAS); then take the exponent; then multiply; and finally subtract.]

There is an important situation when you shouldn't start with what's in the parentheses. Consider the following two examples.

(i) What is the value of $7(100 - 1)$?

Using PEMDAS, you would write $7(100 - 1) = 7(99)$, and then multiply: $7 \times 99 = 693$. But you can do this even quicker in your head if you think of it this way: $7(100 - 1) = 700 - 7 = 693$.

(ii) What is the value of $(77 + 49) \div 7$?

If you followed the rules of PEMDAS, you would first add, $77 + 49 = 126$, and then divide, $126 \div 7 = 18$. This is definitely more difficult and time-consuming than mentally doing $\frac{77}{7} + \frac{49}{7} = 11 + 7 = 18$.

Both of these examples illustrate the very important distributive law.

Key Fact A21

The distributive law

For any real numbers *a*, *b*, and *c*:

- $a(b + c) = ab + ac$
- $a(b - c) = ab - ac$

and if $a \neq 0$

- $\frac{b+c}{a} = \frac{b}{a} + \frac{c}{a}$
- $\frac{b-c}{a} = \frac{b}{a} - \frac{c}{a}$

TIP

Many students who use the distributive law with multiplication forget about it with division. Don't you do that.

EXAMPLE 16

Quantity A	Quantity B
$5(a - 7)$	$5a - 7$

SOLUTION.
By the distributive law, Quantity A = $5a - 35$. The result of subtracting 35 from a number is *always less* than the result of subtracting 7 from that number. Quantity **B** is greater.

EXAMPLE 17

Quantity A	Quantity B
$\frac{50 + x}{5}$	$10 + x$

SOLUTION.

	Quantity A	Quantity B
By the distributive law:	$10 + \frac{x}{5}$	$10 + x$
Subtract 10 from each quantity:	$\frac{x}{5}$	x

The quantities are equal if $x = 0$, but not if $x = 1$.
The answer is **D**.

Inequalities

The number a is ***greater than*** the number b, denoted $a > b$, if a is to the right of b on the number line. Similarly, a is ***less than*** b, denoted $a < b$, if a is to the left of b on the number line. Therefore, if a is positive, $a > 0$, and if a is negative, $a < 0$. Clearly, if $a > b$, then $b < a$.

The following KEY FACT gives an important alternate way to describe greater than and less than.

KEY FACT A22

- **For any numbers a and b:**

  $a > b$ means that $a - b$ is positive.
- **For any numbers a and b:**

  $a < b$ means that $a - b$ is negative.

KEY FACT A23

- **For any numbers a and b, exactly one of the following is true:**

  $a > b$ or $a = b$ or $a < b$.

The symbol ≥ means ***greater than or equal to*** and the symbol ≤ means ***less than or equal to***. The statement "$x \geq 5$" means that x can be 5 or any number greater than 5; the statement "$x \leq 5$" means that x can be 5 or any number less than 5. The statement "$2 < x < 5$" is an abbreviation for the statement "$2 < x$ and $x < 5$." It means that x is a number between 2 and 5 (greater than 2 and less than 5).

Inequalities are very important on the GRE, especially on the quantitative comparison questions where you have to determine which of two quantities is the greater one. KEY FACTS A24 and A25 give some important facts about inequalities.

If the result of performing an arithmetic operation on an inequality is a new inequality in the same direction, we say that the inequality has been ***preserved.*** If the result of performing an arithmetic operation on an inequality is a new inequality in the opposite direction, we say that the inequality has been ***reversed.***

KEY FACT A24

- **Adding a number to an inequality or subtracting a number from an inequality preserves it.**

  If $a < b$, then $a + c < b + c$ and $a - c < b - c$.

  $3 < 7 \Rightarrow 3 + 100 < 7 + 100 \quad (103 < 107)$
  $3 < 7 \Rightarrow 3 - 100 < 7 - 100 \quad (-97 < -93)$
- **Adding inequalities in the same direction preserves them.**

  If $a < b$ and $c < d$, then $a + c < b + d$.

  $3 < 7$ and $5 < 10 \Rightarrow 3 + 5 < 7 + 10 \quad (8 < 17)$
- **Multiplying or dividing an inequality by a positive number preserves it.**

  If $a < b$, and c is positive, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$.

  $3 < 7 \Rightarrow 3 \times 100 < 7 \times 100 \quad (300 < 700)$
  $3 < 7 \Rightarrow 3 \div 100 < 7 \div 100 \quad \left(\frac{3}{100} < \frac{7}{100}\right)$
- **Multiplying or dividing an inequality by a negative number reverses it.**

  If $a < b$, and c is negative, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$.

  $3 < 7 \Rightarrow 3 \times (-100) > 7 \times (-100) \quad (-300 > -700)$
  $3 < 7 \Rightarrow 3 \div (-100) > 7 \div (-100) \quad \left(-\frac{3}{100} > -\frac{7}{100}\right)$
- **Taking negatives reverses an inequality.**

  If $a < b$, then $-a > -b$ and if $a > b$, then $-a < -b$.

  $3 < 7 \Rightarrow -3 > -7$ and $7 > 3 \Rightarrow -7 < -3$
- **If two numbers are each positive or negative, then taking reciprocals reverses an inequality.**

  If a and b are both positive or both negative and $a < b$, then $\frac{1}{a} > \frac{1}{b}$.

  $3 < 7 \Rightarrow \frac{1}{3} > \frac{1}{7} \qquad -7 < -3 \Rightarrow -\frac{1}{7} > -\frac{1}{3}$

KEY FACT A25

Important inequalities for numbers between 0 and 1.

- **If $0 < x < 1$, and a is positive, then $xa < a$. For example: $.85 \times 19 < 19$.**
- **If $0 < x < 1$, and m and n are positive integers with $m > n$, then**

  $x^m < x^n < x$. For example, $\left(\frac{1}{2}\right)^5 < \left(\frac{1}{2}\right)^2 < \frac{1}{2}$.

TIP

Be sure you understand KEY FACT A24; it is very useful. Also, review the important properties listed in KEY FACTS A25 and A26. These properties come up often on the GRE.

- If $0 < x < 1$, then $\sqrt{x} > x$. For example, $\sqrt{\frac{3}{4}} > \frac{3}{4}$.
- If $0 < x < 1$, then $\frac{1}{x} > x$. In fact, $\frac{1}{x} > 1$. For example, $\frac{1}{0.2} > 1 > 0.2$.

KEY FACT A26

Properties of Zero

- **0 is the only number that is neither positive nor negative.**
- **0 is smaller than every positive number and greater than every negative number.**
- **0 is an even integer.**
- **0 is a multiple of every integer.**
- **For every number a: $a + 0 = a$ and $a - 0 = a$.**
- **For every number a: $a \times 0 = 0$.**
- **For every positive integer n: $0^n = 0$.**
- **For every number a (including 0): $a \div 0$ and $\frac{a}{0}$ are *meaningless symbols.* (They are *undefined.*)**
- **For every number a other than 0: $0 \div a = \frac{0}{a} = 0$.**
- **0 is the only number that is equal to its opposite: $0 = -0$.**
- **If the product of two or more numbers is 0, at least one of them is 0.**

Key Fact A27

Properties of 1

- **For any number a: $1 \times a = a$ and $\frac{a}{1} = a$.**
- **For any integer n: $1^n = 1$.**
- **1 is a divisor of every integer.**
- **1 is the smallest positive integer.**
- **1 is an odd integer.**
- **1 is *not* a prime.**

Practice Exercises—Basic Arithmetic

Discrete Quantitative Questions

1. For how many positive integers, a, is it true that $a^2 \leq 2a$?

   Ⓐ None
   Ⓑ 1
   Ⓒ 2
   Ⓓ 4
   Ⓔ More than 4

2. If $0 < a < b < 1$, which of the following statements are true?

   Indicate *all* such statements.

   [A] $a - b$ is negative
   [B] $\frac{1}{ab}$ is positive
   [C] $\frac{1}{b} - \frac{1}{a}$ is positive

3. If the product of 4 consecutive integers is equal to one of them, what is the largest possible value of one of the integers?

   [ ]

4. At 3:00 A.M. the temperature was 13° below zero. By noon it had risen to 32°. What was the average hourly increase in temperature?

   Ⓐ $\left(\frac{19}{9}\right)^\circ$
   Ⓑ $\left(\frac{19}{6}\right)^\circ$
   Ⓒ 5°
   Ⓓ 7.5°
   Ⓔ 45°

5. If a and b are negative, and c is positive, which of the following statements are true? Indicate *all* such statements.

   [A] $a - b < a - c$
   [B] If $a < b$, then $\frac{a}{c} < \frac{b}{c}$
   [C] $\frac{1}{b} < \frac{1}{c}$

6. If $-7 \leq x \leq 7$ and $0 \leq y \leq 12$, what is the greatest possible value of $y - x$?

   Ⓐ −19
   Ⓑ 5
   Ⓒ 7
   Ⓓ 17
   Ⓔ 19

7. If $(7^a)(7^b) = \frac{7^c}{7^d}$, what is d in terms of a, b, and c?

   Ⓐ $\frac{c}{ab}$
   Ⓑ $c - a - b$
   Ⓒ $a + b - c$
   Ⓓ $c - ab$
   Ⓔ $\frac{c}{a + b}$

8. If each of ★ and ❖ can be replaced by +, −, or ×, how many different values are there for the expression 2 ★ 2 ❖ 2?

   [ ]

9. A number is "terrific" if it is a multiple of 2 or 3. How many terrific numbers are there between −11 and 11?
   Ⓐ 6
   Ⓑ 7
   Ⓒ 11
   Ⓓ 15
   Ⓔ 17

10. If x ✪ y represents the number of integers greater than x and less than y, what is the value of $-\pi$ ✪ $\sqrt{2}$?
   Ⓐ 2
   Ⓑ 3
   Ⓒ 4
   Ⓓ 5
   Ⓔ 6

Questions 11 and 12 refer to the following definition.

For any positive integer n, $\tau(n)$ represents the number of positive divisors of n.

11. Which of the following statements are true? Indicate *all* such statements.
   [A] $\tau(5) = \tau(7)$
   [B] $\tau(5) \cdot \tau(7) = \tau(35)$
   [C] $\tau(5) + \tau(7) = \tau(12)$

12. What is the value of $\tau(\tau(\tau(12)))$?
   Ⓐ 1
   Ⓑ 2
   Ⓒ 3
   Ⓓ 4
   Ⓔ 6

13. If p and q are primes greater than 2, which of the following statements must be true? Indicate *all* such statements.
   [A] $p + q$ is even
   [B] pq is odd
   [C] $p^2 - q^2$ is even

14. If $0 < x < 1$, which of the following lists the numbers in increasing order?
   Ⓐ $\sqrt{x}, x, x^2$
   Ⓑ $x^2, x, \sqrt{x}$
   Ⓒ $x^2, \sqrt{x}, x$
   Ⓓ $x, x^2, \sqrt{x}$
   Ⓔ $x, \sqrt{x}, x^2$

15. Which of the following is equal to $(7^8 \times 7^9)^{10}$?
   Ⓐ 7^{27}
   Ⓑ 7^{82}
   Ⓒ 7^{170}
   Ⓓ 49^{170}
   Ⓔ 49^{720}

Quantitative Comparison Questions

Ⓐ Quantity A is greater.
Ⓑ Quantity B is greater.
Ⓒ Quantities A and B are equal.
Ⓓ It is impossible to determine which quantity is greater.

	Quantity A	Quantity B
16.	The product of the odd integers between −8 and 8	The product of the even integers between −9 and 9

a and b are nonzero integers

	Quantity A	Quantity B
17.	$a + b$	ab

	Quantity A	Quantity B
18.	The remainder when a positive integer is divided by 7	7

	Quantity A	Quantity B
19.	$24 \div 6 \times 4$	12

	Quantity A	Quantity B
20.	$\frac{2x - 17}{2}$	$x - 17$

n is an integer greater than 1 that leaves a remainder of 1 when it is divided by 2, 3, 4, 5, and 6

	Quantity A	Quantity B
21.	n	60

	Quantity A	Quantity B
22.	The number of primes that are divisible by 2	The number of primes that are divisible by 3

n is a positive integer

	Quantity A	Quantity B
23.	The number of different prime factors of n	The number of different prime factors of n^2

	Quantity A	Quantity B
24.	The number of even positive factors of 30	The number of odd positive factors of 30

n is a positive integer

	Quantity A	Quantity B
25.	$(-10)^n$	$(-10)^{n+1}$

ANSWER KEY

1. **C**	6. **E**	11. **A, B**	16. **A**	21. **A**
2. **A, B**	7. **B**	12. **C**	17. **D**	22. **C**
3. **3**	8. **4**	13. **A, B, C**	18. **B**	23. **C**
4. **C**	9. **D**	14. **B**	19. **A**	24. **C**
5. **B, C**	10. **D**	15. **C**	20. **A**	25. **D**

ANSWER EXPLANATIONS

1. **(C)** Since a is positive, we can divide both sides of the given inequality by a: $a^2 \le 2a \Rightarrow a \le 2 \Rightarrow a = 1$ or 2.

2. **(A)(B)** Since $a < b$, $a - b$ is negative (A is true). Since a and b are positive, so is their product, ab; and the reciprocal of a positive number is positive (B is true).
$\frac{1}{b} - \frac{1}{a} = \frac{a-b}{ab}$, and we have just seen that the numerator is negative and the denominator positive; so the value of the fraction is negative (C is false).

3. **3** If all four integers were negative, their product would be positive, and so could not equal one of them. If all of the integers were positive, their product would be much greater than any of them (even $1 \times 2 \times 3 \times 4 = 24$). So, the integers must include 0, in which case their product *is* 0. The largest set of four consecutive integers that includes 0 is 0, 1, 2, 3.

4. **(C)** In the 9 hours from 3:00 to 12:00, the temperature rose $32 - (-13) = 32 + 13 = 45$ degrees. So, the average hourly increase was $45° \div 9 = 5°$.

5. **(B)(C)** Since b is negative and c is positive, $b < c \Rightarrow -b > -c \Rightarrow a - b > a - c$ (A is false). Since c is positive, dividing by c preserves the inequality. (B is true.) Since b is negative, $\frac{1}{b}$ is negative, and so is less than $\frac{1}{c}$, which is positive (C is true).

6. **(E)** To make $y - x$ as large as possible, let y be as big as possible (12), and subtract the smallest amount possible ($x = -7$): $12 - (-7) = 19$.

7. **(B)** $(7^a)(7^b) = 7^{a+b}$, and $\frac{7^c}{7^d} = 7^{c-d}$. Therefore,
$a + b = c - d \Rightarrow a + b + d = c \Rightarrow d = c - a - b$

8. **4** Just list the 9 possible outcomes of replacing ★ and ❖ by +, –, and ×, and see that there are 4 different values: –2, 2, 6, 8.

$2 + 2 + 2 = 6$	$2 - 2 - 2 = -2$	$2 \times 2 \times 2 = 8$
$2 + 2 - 2 = 2$	$2 - 2 \times 2 = -2$	$2 \times 2 + 2 = 6$
$2 + 2 \times 2 = 6$	$2 - 2 + 2 = 2$	$2 \times 2 - 2 = 2$

9. **(D)** There are 15 "terrific" numbers: 2, 3, 4, 6, 8, 9, 10, their opposites, and 0.

10. **(D)** There are 5 integers (1, 0, –1, –2, –3) that are greater than –3.14 ($-\pi$) and less than 1.41 $(\sqrt{2})$.

11. **(A)(B)** Since 5 and 7 have two positive factors each, $\tau(5) = \tau(7)$. (A is true.) Since 35 has 4 divisors (1, 5, 7, and 35) and $\tau(5) \cdot \tau(7) = 2 \times 2 = 4$. (B is true.) Since the positive divisors of 12 are 1, 2, 3, 4, 6, and 12, $\tau(12)$ is 6, which is *not* equal to 2 + 2. (C is false.)

12. **(C)** $\tau(\tau(\tau(12))) = \tau(\tau(6)) = \tau(4) = 3$

13. **(A)(B)(C)** All primes greater than 2 are odd, so p and q are odd, and $p + q$, the sum of two odd numbers, is even (A is true). The product of two odd numbers is odd (B is true). Since p and q are odd, so are their squares, and so the difference of their squares is even (C is true).

14. **(B)** For any number, x, between 0 and 1: $x^2 < x$ and $x < \sqrt{x}$.

15. **(C)** First, multiply inside the parentheses: $7^8 \times 7^9 = 7^{17}$; then, raise to the 10th power: $(7^{17})^{10} = 7^{170}$.

16. **(A)** Since Quantity A has 4 negative factors (–7, –5, –3, –1), it is positive. Quantity B also has 4 negative factors, but be careful—it also has the factor 0, and so Quantity B is 0.

17. **(D)** If a and b are each 1, then $a + b = 2$, and $ab = 1$; so, Quantity A is greater. But, if a and b are each 3, $a + b = 6$, and $ab = 9$, then Quantity B is greater.

18. **(B)** The remainder is *always* less than the divisor.

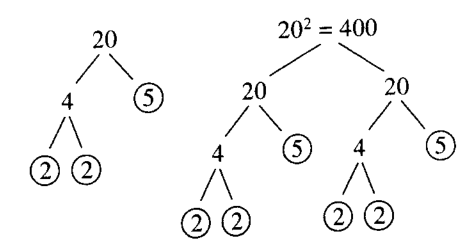
19. **(A)** According to PEMDAS, you divide and multiply from left to right (do *not* do the multiplication first): $24 \div 6 \times 4 = 4 \times 4 = 16$.

20. **(A)** By the distributive law, $\frac{2x-17}{2} = \frac{2x}{2} - \frac{17}{2} = x - 8.5$, which is greater than $x - 17$ (the larger the number you subtract, the smaller the difference.)

21. **(A)** The LCM of 2, 3, 4, 5, 6 is 60; and all multiples of 60 are divisible by each of them. So, n could be 61 or 1 more than any multiple of 60.

22. **(C)** The only prime divisible by 2 is 2, and the only prime divisible by 3 is 3. Quantity A and Quantity B are each 1.

23. **(C)** If you make a factor tree for n^2, the first branches would be n and n. Now, when you factor each n, you get exactly the same prime factors. (See the example below.)



24. **(C)** Just list the factors of 30: 1, 2, 3, 5, 6, 10, 15, 30. Four of them are odd and four are even.

25. **(D)** If n is even, then $n + 1$ is odd, and consequently $(-10)^n$ is positive, whereas $(-10)^{n+1}$ is negative. If n is odd, exactly the opposite is true.

11-B. FRACTIONS AND DECIMALS

Several questions on the GRE involve fractions or decimals. The KEY FACTS in this section cover all of the important facts you need to know for the GRE.

When a whole is *divided* into n equal parts, each part is called *one-nth* of the whole, written $\frac{1}{n}$. For example, if a pizza is cut (*divided*) into 8 equal slices, each slice is one-eighth $\left(\frac{1}{8}\right)$ of the pizza; a day is *divided* into 24 equal hours, so an hour is one-twenty-fourth $\left(\frac{1}{24}\right)$ of a day; and an inch is one-twelfth $\left(\frac{1}{12}\right)$ of a foot.

- If Donna slept for 5 hours, she slept for five-twenty-fourths $\left(\frac{5}{24}\right)$ of a day.
- If Taryn bought 8 slices of pizza, she bought eight-eighths $\left(\frac{8}{8}\right)$ of a pie.
- If Aviva's shelf is 30 inches long, it measures thirty-twelfths $\left(\frac{30}{12}\right)$ of a foot.

Numbers such as $\frac{5}{24}$, $\frac{8}{8}$, and $\frac{30}{12}$, in which one integer is written over a second integer, are called ***fractions.*** The center line is called the fraction bar. The number above the bar is called the ***numerator***, and the number below the bar is called the ***denominator***.

> **CAUTION**
> The denominator of a fraction can *never* be 0.

- A fraction, such as $\frac{5}{24}$, in which the numerator is less than the denominator, is called a ***proper fraction.*** Its value is less than 1.
- A fraction, such as $\frac{30}{12}$, in which the numerator is more than the denominator, is called an ***improper fraction.*** Its value is greater than 1.
- A fraction, such as $\frac{8}{8}$, in which the numerator and denominator are the same, is also ***improper***, but it is equal to 1.

It is useful to think of the fraction bar as a symbol for division. If three pizzas are divided equally among eight people, each person gets $\frac{3}{8}$ of a pizza. If you actually divide 3 by 8, you get that $\frac{3}{8} = 0.375$.

KEY FACT B1

Every fraction, proper or improper, can be expressed in decimal form (or as a whole number) by dividing the numerator by the denominator.

$\frac{3}{10} = 0.3$ $\quad \frac{3}{4} = 0.75$ $\quad \frac{5}{8} = 0.625$ $\quad \frac{3}{16} = 0.1875$

$\frac{8}{8} = 1$ $\quad \frac{11}{8} = 1.375$ $\quad \frac{48}{16} = 3$ $\quad \frac{100}{8} = 12.5$

Note that any number beginning with a decimal point can be written with a 0 to the left of the decimal point. In fact, some calculators will express 3 ÷ 8 as .375, whereas others will print 0.375.

Unlike the examples above, when most fractions are converted to decimals, the division does not terminate after 2 or 3 or 4 decimal places; rather it goes on forever with some set of digits repeating itself.

$\frac{2}{3} = 0.666666...$ $\quad \frac{3}{11} = 0.272727...$ $\quad \frac{5}{12} = 0.416666...$ $\quad -\frac{17}{15} = -1.133333...$

A convenient way to represent repeating decimals is to place a bar over the digits that repeat. For example, the decimal equivalent of the four fractions, above, could be written as follows:

$$\frac{2}{3} = 0.\overline{6} \quad \frac{3}{11} = 0.\overline{27} \quad \frac{5}{12} = 0.41\overline{6} \quad -\frac{17}{15} = -1.1\overline{3}$$

A ***rational number*** is any number that can be expressed as a fraction, $\frac{a}{b}$, where a and b are integers. For example, 2, –2.2, and $2\frac{1}{2}$ are all rational since they can be expressed as $\frac{2}{1}$, $\frac{-22}{10}$, and $\frac{5}{2}$, respectively. When written as decimals, all rational numbers either terminate or repeat. Numbers such as $\sqrt{2}$ and π that cannot be expressed as fractions whose numerators and denominators are integers are called ***irrational numbers.*** The decimal expansions of irrational numbers neither terminate nor repeat. For example, $\sqrt{2} = 1.414213562.....$ and $\pi = 3.141592654 \ldots$.

Comparing Fractions and Decimals

KEY FACT B2

To compare two positive decimals, follow these rules.

- **Whichever number has the greater number to the left of the decimal point is greater: since 11 > 9, 11.001 > 9.896; since 1 > 0, 1.234 > 0.8; and since 3 > –3, 3.01 > –3.95. (Recall that if a decimal is written without a number to the left of the decimal point, you may assume that a 0 is there. So, 1.234 > 0.8.)**

- **If the numbers to the left of the decimal point are equal (or if there are no numbers to the left of the decimal point), proceed as follows:**
  1. **If the numbers do not have the same number of digits to the right of the decimal point, add zeros to the end of the shorter one.**
  2. **Now, compare the numbers *ignoring* the decimal point.**

For example, to compare 1.83 and 1.823, add a 0 to the end of 1.83, forming 1.830. Now compare them, *thinking of them as whole numbers*: since, 1830 > 1823, then 1.830 > 1.823.

EXAMPLE 1

Quantity A	Quantity B
.2139	.239

SOLUTION.
Do not think that Quantity A is greater because 2139 > 239. Be sure to add a 0 to the end of 0.239 (forming 0.2390) before comparing. Now, since 2390 > 2139, Quantity **B** is greater.

KEY FACT B3

There are two methods of comparing positive fractions:

1. **Convert them to decimals (by dividing), and use KEY FACT B2.**
2. **Cross-multiply.**

For example, to compare $\frac{1}{3}$ and $\frac{3}{8}$, we have two choices.

1. Write $\frac{1}{3}$ = .3333... and $\frac{3}{8}$ = .375. Since .375 > .333, then $\frac{3}{8} > \frac{1}{3}$.

2. Cross-multiply: $\frac{1}{3} \times \frac{3}{8}$. Since 3 × 3 > 8 × 1, then $\frac{3}{8} > \frac{1}{3}$.

KEY FACT B4

When comparing positive fractions, there are three situations in which it is easier just to look at the fractions, and not use either method in KEY FACT B3.

1. **If the fractions have the same denominator, the fraction with the larger numerator is greater. Just as $9 is more than $7, and 9 books are more than 7 books, 9 fortieths are more than 7 fortieths: $\frac{9}{40} > \frac{7}{40}$.**
2. **If the fractions have the same numerator, the fraction with the smaller denominator is greater.**

If you divide a cake into 5 equal pieces, each piece is larger than the pieces you would get if you had divided the cake into 10 equal pieces: $\frac{1}{5} > \frac{1}{10}$, and similarly $\frac{3}{5} > \frac{3}{10}$.

3. **Sometimes the fractions are so familiar or easy to work with, you just know the answer. For example, $\frac{3}{4} > \frac{1}{5}$ and $\frac{11}{20} > \frac{1}{2}$ $\left(\text{since } \frac{10}{20} = \frac{1}{2}\right)$.**

KEY FACTS B2, B3, and B4 apply to *positive* decimals and fractions.

KEY FACT B5

- **Clearly, any positive number is greater than any negative number:**

$$\frac{1}{2} > -\frac{1}{5} \quad \text{and} \quad 0.123 > -2.56$$

- **For negative decimals and fractions, use KEY FACT A24, which states that if $a > b$, then $-a < -b$:**

$$\frac{1}{2} > \frac{1}{5} \Rightarrow -\frac{1}{2} < -\frac{1}{5} \quad \text{and} \quad 0.83 > 0.829 \Rightarrow -0.83 < -0.829$$

EXAMPLE 2

Which of the following lists the fractions $\frac{2}{3}$, $\frac{5}{8}$, and $\frac{13}{20}$ in order from least to greatest?

Ⓐ $\frac{2}{3}, \frac{5}{8}, \frac{13}{20}$ Ⓑ $\frac{5}{8}, \frac{2}{3}, \frac{13}{20}$ Ⓒ $\frac{5}{8}, \frac{13}{20}, \frac{2}{3}$ Ⓓ $\frac{13}{20}, \frac{5}{8}, \frac{2}{3}$ Ⓔ $\frac{13}{20}, \frac{2}{3}, \frac{5}{8}$

SOLUTION.
Use your calculator to quickly convert each to a decimal, writing down the first few decimal places: $\frac{2}{3}$ = 0.666, $\frac{5}{8}$ = 0.625, and $\frac{13}{20}$ = 0.65. It is now easy to order the decimals: 0.625 < 0.650 < 0.666. The answer is **C**.

ALTERNATIVE SOLUTION.
Cross-multiply.

- $\frac{2}{3} > \frac{5}{8}$ since 8 × 2 > 3 × 5.
- $\frac{13}{20} > \frac{5}{8}$ since 8 × 13 > 20 × 5.
- $\frac{2}{3} > \frac{13}{20}$ since 20 × 2 > 3 × 13.

EXAMPLE 3

$0 < x < y$

Quantity A	Quantity B
$\frac{1}{x} - \frac{1}{y}$	0

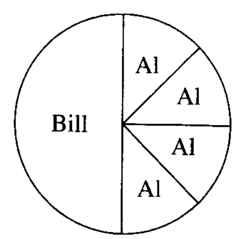
SOLUTION.

By KEY FACT B4, $x < y \Rightarrow \frac{1}{x} > \frac{1}{y}$, and so by KEY FACT A22, $\frac{1}{x} - \frac{1}{y}$ is positive. Quantity **A** is greater.

Equivalent Fractions

If Bill and Al shared a pizza, and Bill ate $\frac{1}{2}$ the pizza and Al ate $\frac{4}{8}$ of it, they had exactly the same amount.

We express this idea by saying that $\frac{1}{2}$ and $\frac{4}{8}$ are ***equivalent fractions***: they have the exact same value.



NOTE: If you multiply both the numerator and denominator of $\frac{1}{2}$ by 4 you get $\frac{4}{8}$; and if you divide both the numerator and denominator of $\frac{4}{8}$ by 4 you get $\frac{1}{2}$. This illustrates the next KEY FACT.

KEY FACT B6

Two fractions are equivalent if multiplying or dividing both the numerator and denominator of the first one *by the same number* gives the second one.

Consider the following two cases.

1. When the numerator and denominator of $\frac{3}{8}$ are each multiplied by 15, the products are $3 \times 15 = 45$ and $8 \times 15 = 120$. Therefore, $\frac{3}{8}$ and $\frac{45}{120}$ are equivalent fractions.
2. $\frac{2}{3}$ and $\frac{28}{45}$ are not equivalent fractions because 2 must be multiplied by 14 to get 28, but 3 must be multiplied by 15 to get 45.

KEY FACT B7

To determine if two fractions are equivalent, cross-multiply. The fractions are equivalent if and only if the two products are equal.

For example, since $120 \times 3 = 8 \times 45$, then $\frac{3}{8}$ and $\frac{45}{120}$ are equivalent.

Since $45 \times 2 \neq 3 \times 28$, then $\frac{2}{3}$ and $\frac{28}{45}$ are not equivalent fractions.

A fraction is in ***lowest terms*** if no positive integer greater than 1 is a factor of both the numerator and denominator. For example, $\frac{9}{20}$ is in lowest terms, since no integer greater than 1 is a factor of both 9 and 20; but $\frac{9}{24}$ is not in lowest terms, since 3 is a factor of both 9 and 24.

KEY FACT B8

Every fraction can be *reduced* to lowest terms by dividing the numerator and the denominator by their greatest common factor (GCF). If the GCF is 1, the fraction is already in lowest terms.

For any positive integer *n*: *n*!, read *n factorial*, is the product of all the integers from 1 to *n*, inclusive.

EXAMPLE 4

What is the value of $\frac{6!}{8!}$?

Ⓐ $\frac{1}{56}$ Ⓑ $\frac{1}{48}$ Ⓒ $\frac{1}{8}$ Ⓓ $\frac{1}{4}$ Ⓔ $\frac{3}{4}$

SOLUTION.

Even with a calculator, you do not want to calculate 6! (1·2·3·4·5·6 = 720) and 8! (1·2·3·4·5·6·7·8 = 40,320) and then take the time to reduce $\frac{720}{40{,}320}$. Here's the easy solution:

$$\frac{6!}{8!} = \frac{\overset{1}{\cancel{6\times5\times4\times3\times2\times1}}}{8\times7\times\underset{1}{\cancel{6\times5\times4\times3\times2\times1}}} = \frac{1}{8\times7} = \frac{1}{56}$$

Arithmetic Operations with Decimals

Arithmetic operations with decimals should be done on your calculator, unless they are so easy that you can do them in your head.

Multiplying and dividing by powers of 10 is particularly easy and does not require a calculator: they can be accomplished just by moving the decimal point.

KEY FACT B9

To multiply any decimal or whole number by a power of 10, move the decimal point as many places to the *right* as there are 0s in the power of 10, filling in with 0s, if necessary.

$1.35 \times 10 = 13.5$ 1 $\qquad 1.35 \times 100 = 135$ 2

$1.35 \times 1000 = 1350$ 3

$23 \times 10 = 230$ 1 $\qquad 23 \times 100 = 2300$ 2

$23 \times 1{,}000{,}000 = 23{,}000{,}000$ 6

KEY FACT B10

To divide any decimal or whole number by a power of 10, move the decimal point as many places to the *left* as there are 0s in the power of 10, filling in with 0s, if necessary.

$67.8 \div 10 = 6.78$ 1 $\qquad 67.8 \div 100 = 0.678$ 2

$67.8 \div 1000 = 0.0678$ 3

$14 \div 10 = 1.4$ 1 $\qquad 14 \div 100 = 0.14$ 2

$14 \div 1{,}000{,}000 = 0.000014$ 6

EXAMPLE 5

Quantity A	Quantity B
3.75×10^4	$37{,}500{,}000 \div 10^3$

SOLUTION.
To evaluate Quantity A, move the decimal point 4 places to the right: **37,500**. To evaluate Quantity B, move the decimal point 3 places to the left: **37,500**. The answer is **C**.

Arithmetic Operations with Fractions

KEY FACT B11

To multiply two fractions, multiply their numerators and multiply their denominators:

$$\frac{3}{5} \times \frac{4}{7} = \frac{3 \times 4}{5 \times 7} = \frac{12}{35} \qquad \frac{3}{5} \times \frac{\pi}{2} = \frac{3 \times \pi}{5 \times 2} = \frac{3\pi}{10}$$

KEY FACT B12

To multiply a fraction by any other number, write that number as a fraction whose denominator is 1:

$$\frac{3}{5} \times 7 = \frac{3}{5} \times \frac{7}{1} = \frac{21}{5} \qquad \frac{3}{5} \times \pi = \frac{3}{5} \times \frac{\pi}{1} = \frac{3\pi}{5}$$

TACTIC B1

Before multiplying fractions, reduce. You may reduce by dividing any numerator and any denominator by a common factor.

EXAMPLE 6

Express the product, $\frac{3}{4} \times \frac{8}{9} \times \frac{15}{16}$, in lowest terms.

SOLUTION.
You could use your calculator to multiply the numerators and denominators: $\frac{360}{576}$.

It is better, however, to use TACTIC B1 and reduce first:

$$\frac{\overset{1}{\cancel{3}}}{4} \times \frac{\overset{1}{\cancel{8}}}{\underset{1}{\cancel{\underset{3}{\cancel{9}}}}} \times \frac{\overset{5}{\cancel{15}}}{\underset{2}{\cancel{16}}} = \frac{1 \times 1 \times 5}{4 \times 1 \times 2} = \frac{5}{8}$$

TACTIC

B2

When a problem requires you to find a fraction of a number, multiply.

EXAMPLE 7

If $\frac{4}{7}$ of the 350 sophomores at Monroe High School are girls, and $\frac{7}{8}$ of them play on a team, how many sophomore girls do not play on a team?

SOLUTION.

There are $\frac{4}{7} \times 350 = 200$ sophomore girls.

Of these, $\frac{7}{8} \times 200 = 175$ play on a team. So, $200 - 175 = \mathbf{25}$ do not play on a team.

The ***reciprocal*** of any nonzero number x is that number y such that $xy = 1$. Since $x\left(\frac{1}{x}\right) = 1$, then $\frac{1}{x}$ is the reciprocal of x. Similarly, the reciprocal of the fraction $\frac{a}{b}$ is the fraction $\frac{b}{a}$, since $\frac{a}{b} \cdot \frac{b}{a} = 1$.

KEY FACT B13

To divide any number by a fraction, multiply that number by the reciprocal of the fraction.

$$20 \div \frac{2}{3} = \frac{20}{1} \times \frac{3}{2} = 30 \qquad \frac{3}{5} \div \frac{2}{3} = \frac{3}{5} \times \frac{3}{2} = \frac{9}{10}$$

$$\sqrt{2} \div \frac{2}{3} = \frac{\sqrt{2}}{1} \times \frac{3}{2} = \frac{3\sqrt{2}}{2} \qquad \frac{\pi}{5} \div \frac{2}{3} = \frac{\pi}{5} \times \frac{3}{2} = \frac{3\pi}{10}$$

EXAMPLE 8

In the meat department of a supermarket, 100 pounds of chopped meat was divided into packages, each of which weighed $\frac{4}{7}$ of a pound. How many packages were there?

SOLUTION.

$100 \div \frac{4}{7} = \frac{100}{1} \times \frac{7}{4} = \mathbf{175}$

KEY FACT B14

- **To add or subtract fractions with the same denominator, add or subtract the numerators and keep the denominator:**

$$\frac{4}{9} + \frac{1}{9} = \frac{5}{9} \quad \textbf{and} \quad \frac{4}{9} - \frac{1}{9} = \frac{3}{9} = \frac{1}{3}$$

- **To add or subtract fractions with different denominators, first rewrite the fractions as equivalent fractions with the same denominators:**

$$\frac{1}{6} + \frac{3}{4} = \frac{2}{12} + \frac{9}{12} = \frac{11}{12}$$

NOTE: The *easiest* common denominator to find is the product of the denominators ($6 \times 4 = 24$, in this example), but the best denominator to use is the ***least common denominator***, which is the least common multiple (LCM) of the denominators (12, in this case). Using the least common denominator minimizes the amount of reducing that is necessary to express the answer in lowest terms.

KEY FACT B15

If $\frac{a}{b}$ is the fraction of a whole that satisfies some property, then $1 - \frac{a}{b}$ is the fraction of that whole that does not satisfy it.

EXAMPLE 9

In a jar, $\frac{1}{2}$ of the marbles are red, $\frac{1}{4}$ are white, and $\frac{1}{5}$ are blue. What fraction of the marbles are neither red, white, nor blue?

SOLUTION.
The red, white, and blue marbles constitute

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{5}=\frac{10}{20}+\frac{5}{20}+\frac{4}{20}=\frac{19}{20}$$

of the total, so $1-\frac{19}{20}=\frac{20}{20}-\frac{19}{20}=\frac{1}{20}$ of the marbles are neither red, white, nor blue.

Alternatively, you could convert the fractions to decimals and use your calculator.

$$0.5+0.25+0.2=0.95$$
$$1-0.95=0.05=\frac{5}{100}$$

Remember, on the GRE you do not have to reduce fractions, so $\frac{5}{100}$ is an acceptable answer.

EXAMPLE 10

Lindsay ate $\frac{1}{3}$ of a cake and Emily ate $\frac{1}{4}$ of it. What fraction of the cake was still uneaten?

SOLUTION.
$\frac{1}{3}+\frac{1}{4}=\frac{4}{12}+\frac{3}{12}=\frac{7}{12}$ of the cake was eaten, and $1-\frac{7}{12}=\frac{5}{12}$ was uneaten.

EXAMPLE 11

Lindsay ate $\frac{1}{3}$ of a cake and Emily ate $\frac{1}{4}$ of what was left. What fraction of the cake was still uneaten?

CAUTION: Be sure to read questions carefully. In Example 10, Emily ate $\frac{1}{4}$ of the cake. In Example 11, however, she only ate $\frac{1}{4}$ of the $\frac{2}{3}$ that was left after Lindsay had her piece: she ate $\frac{1}{4}\times\frac{2}{3}=\frac{1}{6}$ of the cake.

SOLUTION.
$\frac{1}{3}+\frac{1}{6}=\frac{2}{6}+\frac{1}{6}=\frac{1}{2}$ of the cake was eaten, and the other $\frac{1}{2}$ was uneaten.

Arithmetic Operations with Mixed Numbers

A ***mixed number*** is a number such as $3\frac{1}{2}$, which consists of an integer followed by a fraction. It is an abbreviation for the *sum* of the number and the fraction; so, $3\frac{1}{2}$ is an abbreviation for $3+\frac{1}{2}$. Every mixed number can be written as an improper fraction, and every improper fraction can be written as a mixed number:

$$3\frac{1}{2}=3+\frac{1}{2}=\frac{3}{1}+\frac{1}{2}=\frac{6}{2}+\frac{1}{2}=\frac{7}{2} \quad \text{and} \quad \frac{7}{2}=\frac{6}{2}+\frac{1}{2}=3+\frac{1}{2}=3\frac{1}{2}$$

On the GRE you should perform all arithmetic operations on mixed numbers in one of the following two ways:

- Change the mixed numbers to improper fractions and use the rules you already know for performing arithmetic operations on fractions.
- Change the mixed numbers to decimals and perform the arithmetic on your calculator.

CAUTION

A common mistake for many students is to think that

$3\times5\frac{1}{2}$ is $15\frac{1}{2}$— it isn't!

If you need to multiply $3\times5\frac{1}{2}$, use one of the two methods mentioned above.

- $3\times5\frac{1}{2}=3\left(5+\frac{1}{2}\right)=15+\frac{3}{2}=15+1\frac{1}{2}=16\frac{1}{2}$
- $3\times5\frac{1}{2}=3\times5.5=16.5=16\frac{1}{2}$

Complex Fractions

A ***complex fraction*** is a fraction, such as $\dfrac{1+\frac{1}{6}}{2-\frac{3}{4}}$, which has one or more fractions in its numerator or denominator or both.

KEY FACT B16

There are two ways to simplify a complex fraction:

- **Multiply *every* term in the numerator and denominator by the least common multiple of all the denominators that appear in the fraction.**
- **Simplify the numerator and the denominator, and then divide.**

To simplify $\dfrac{1+\frac{1}{6}}{2-\frac{3}{4}}$

- either multiply each term by 12, the LCM of 6 and 4:

$$\frac{12(1)+12\left(\frac{1}{6}\right)}{12(2)-12\left(\frac{3}{4}\right)}=\frac{12+2}{24-9}=\frac{14}{15},\text{ or}$$

- simplify the numerator and denominator:

$$\frac{1+\frac{1}{6}}{2-\frac{3}{4}}=\frac{\frac{7}{6}}{\frac{5}{4}}=\frac{7}{6}\times\frac{4}{5}=\frac{14}{15}.$$

Practice Exercises—Fractions and Decimals

Discrete Quantitative Questions

1. A biology class has 12 boys and 18 girls. What fraction of the class are boys?

$$\frac{\square}{\square}$$

2. For how many integers, a, between 30 and 40 is it true that $\frac{5}{a}$, $\frac{8}{a}$, and $\frac{13}{a}$ are all in lowest terms?

Ⓐ 1
Ⓑ 2
Ⓒ 3
Ⓓ 4
Ⓔ 5

3. What fractional part of a week is 98 hours?

$$\frac{\square}{\square}$$

4. What is the value of the product $\frac{5}{5}\times\frac{5}{10}\times\frac{5}{15}\times\frac{5}{20}\times\frac{5}{25}$?

Ⓐ $\frac{1}{120}$
Ⓑ $\frac{1}{60}$
Ⓒ $\frac{1}{30}$
Ⓓ $\frac{5}{30}$
Ⓔ $\frac{1}{2}$

5. If $\frac{3}{11}$ of a number is 22, what is $\frac{6}{11}$ of that number?

Ⓐ 6
Ⓑ 11
Ⓒ 12
Ⓓ 33
Ⓔ 44

6. Jason won some goldfish at the state fair. During the first week, $\frac{1}{5}$ of them died, and during the second week, $\frac{3}{8}$ of those still alive at the end of the first week died. What fraction of the original goldfish were still alive after two weeks?

Ⓐ $\frac{3}{10}$
Ⓑ $\frac{17}{40}$
Ⓒ $\frac{1}{2}$
Ⓓ $\frac{23}{40}$
Ⓔ $\frac{7}{10}$

7. $\frac{5}{8}$ of 24 is equal to $\frac{15}{7}$ of what number?

Ⓐ 7
Ⓑ 8
Ⓒ 15
Ⓓ $\frac{7}{225}$
Ⓔ $\frac{225}{7}$

8. If $7a = 3$ and $3b = 7$, what is the value of $\frac{a}{b}$?

Ⓐ $\frac{9}{49}$

Ⓑ $\frac{3}{7}$

Ⓒ 1

Ⓓ $\frac{7}{3}$

Ⓔ $\frac{49}{9}$

9. What is the value of $\dfrac{\frac{7}{9}\times\frac{7}{9}}{\frac{7}{9}+\frac{7}{9}+\frac{7}{9}}$?

Ⓐ $\frac{7}{27}$

Ⓑ $\frac{2}{3}$

Ⓒ $\frac{7}{9}$

Ⓓ $\frac{9}{7}$

Ⓔ $\frac{3}{2}$

10. Which of the following expressions are greater than x when $x = \frac{9}{11}$?

Indicate *all* such expressions.

[A] $\frac{1}{x}$

[B] $\frac{x+1}{x}$

$x+1$

11. One day at Lincoln High School, $\frac{1}{12}$ of the students were absent, and $\frac{1}{5}$ of those present went on a field trip. If the number of students staying in school that day was 704, how many students are enrolled at Lincoln High?

[ ]

12. If $a = 0.87$, which of the following expressions are less than a?

Indicate *all* such expressions.

[A] $\sqrt{a}$

[B] a^2

[C] $\frac{1}{a}$

13. For what value of x is $\frac{(34.56)(7.89)}{x} = (.3456)(78.9)$?

Ⓐ .001
Ⓑ .01
Ⓒ .1
Ⓓ 10
Ⓔ 100

14. If $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$, and C is the set consisting of all the fractions whose numerators are in A and whose denominators are in B, what is the product of all of the numbers in C?

Ⓐ $\frac{1}{64}$

Ⓑ $\frac{1}{48}$

Ⓒ $\frac{1}{24}$

Ⓓ $\frac{1}{12}$

15. For the final step in a calculation, Ezra accidentally divided by 1000 instead of multiplying by 1000. What should he do to his incorrect answer to correct it?

Ⓐ Multiply it by 1000.
Ⓑ Multiply it by 100,000.
Ⓒ Multiply it by 1,000,000.
Ⓓ Square it.
Ⓔ Double it.

Quantitative Comparison Questions

Ⓐ Quantity A is greater.
Ⓑ Quantity B is greater.
Ⓒ Quantities A and B are equal.
Ⓓ It is impossible to determine which quantity is greater.

	Quantity A	Quantity B
16.	$\frac{5}{13}$ of 47	$\frac{47}{13}$ of 5

$x = -\frac{2}{3}$ and $y = \frac{3}{5}$

	Quantity A	Quantity B
17.	xy	$\frac{x}{y}$

	Quantity A	Quantity B
18.	$\dfrac{15}{\frac{1}{15}}$	1

Judy needed 8 pounds of chicken. At the supermarket, the only packages available weighed $\frac{3}{4}$ of a pound each.

	Quantity A	Quantity B
19.	The number of packages Judy	11

	Quantity A	Quantity B
20.	$\frac{11}{12}$ or $\frac{13}{14}$	$\frac{14}{15}$

$a \nabla b = \frac{a}{b} + \frac{b}{a}$

	Quantity A	Quantity B
21.	$3\nabla 4$	$\frac{1}{2}\nabla\frac{2}{3}$

	Quantity A	Quantity B
22.	$\frac{100}{2^{100}}$	$\frac{100}{3^{100}}$

	Quantity A	Quantity B
23.	$\left(-\frac{1}{2}\right)\left(-\frac{3}{4}\right)\left(-\frac{5}{6}\right)\left(-\frac{7}{8}\right)$	$\left(-\frac{3}{7}\right)\left(-\frac{5}{9}\right)\left(-\frac{7}{11}\right)$

$a = \frac{1}{2}$ and $b = \frac{1}{3}$

	Quantity A	Quantity B
24.	$\frac{a}{b}$	$\frac{b}{a}$

	Quantity A	Quantity B
25.	$\left(\frac{3}{11}\right)^2$	$\sqrt{\frac{3}{11}}$

ANSWER KEY

1. $\frac{2}{5}$	4. **A**	9. **A**	14. **A**	19. **C**	24. **A**
	5. **E**	10. **A, B**	15. **C**	20. **B**	25. **B**
2. **C**	6. **C**	11. **960**	16. **C**	21. **C**	
	7. **A**	12. **B**	17. **A**	22. **A**	
3. $\frac{7}{12}$	8. **A**	13. **D**	18. **A**	23. **A**	

Answer Explanations

1. $\frac{2}{5}$ The class has 30 students, of whom 12 are boys. So, the boys make up $\frac{12}{30} = \frac{5}{2}$ of the class.

2. **(C)** If a is even, then $\frac{8}{a}$ is *not* in lowest terms, since both a and 8 are divisible by 2. Therefore, the only possibilities are 31, 33, 35, 37, and 39; but $\frac{5}{35} = \frac{1}{7}$ and $\frac{13}{39} = \frac{1}{3}$, so only 3 integers—31, 33, and 37—satisfy the given condition.

3. $\frac{7}{12}$ There are 24 hours in a day and 7 days in a week, so there are $24 \times 7 = 168$ hours in a week: $\frac{98}{168} = \frac{7}{12}$.

4. **(A)** Reduce each fraction and multiply:

$$1 \times \frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} \times \frac{1}{5} = \frac{1}{120}.$$

5. **(E)** Don't bother writing an equation for this one; just think. We know that $\frac{3}{11}$ of the number is 22, and $\frac{6}{11}$ of a number is twice as much as $\frac{3}{11}$ of it: $2 \times 22 = 44$.

6. **(C)** The *algebra* way is to let x = the number of goldfish Jason won. During the first week $\frac{1}{5}x$ died, so $\frac{4}{5}x$ were still alive. During week two, $\frac{3}{8}$ of them died and $\frac{5}{8}$ of them survived:

$$\left(\frac{\cancel{5}^{1}}{\cancel{8}_{2}}\right)\left(\frac{\cancel{4}^{1}}{\cancel{5}_{1}}x\right) = \frac{1}{2}x.$$

On the GRE, the best way is to assume that the original number of goldfish was 40, the LCM of the denominators (see TACTIC 3, Chapter 9). Then, 8 died the first week $\left(\frac{1}{5} \text{ of } 40\right)$, and 12 of the 32 survivors $\left(\frac{3}{8} \text{ of } 32\right)$ died the second week. In all, 8 + 12 = 20 died; the other 20 $\left(\frac{1}{2}\right.$ the original number$\left.\right)$ were still alive.

7. **(A)** If x is the number, then $\frac{15}{7}x = \frac{5}{\cancel{8}_{1}} \times \cancel{24}^{3} = 15$. So, $\frac{15}{7}x = 15$, which means (dividing by 15) that $\frac{1}{7}x = 1$, and so $x = 7$.

8. **(A)** $7a = 3$ and $3b = 7 \Rightarrow a = \frac{3}{7}$ and $b = \frac{7}{3} \Rightarrow \frac{a}{b} = \frac{3}{7} \div \frac{7}{3} = \frac{3}{7} \times \frac{3}{7} = \frac{9}{49}$.

9. **(A)** Don't start by doing the arithmetic. This is just $\frac{(a)(a)}{a+a+a} = \frac{(a)(\cancel{a})}{3\cancel{a}} = \frac{a}{3}$.

Now, replacing a with $\frac{7}{9}$ gives $\frac{7}{9} \div 3 = \frac{7}{9} \times \frac{1}{3} = \frac{7}{27}$.

10. **(A)(B)** The reciprocal of a positive number less than 1 is greater than 1 (A is true). $\frac{x+1}{x} = 1 + \frac{1}{x}$, which is greater than 1 (B is true). Since $\frac{9}{11} + 1$ is positive and $\frac{9}{11} - 1$ is negative, when $x = \frac{9}{11}$, $\frac{x+1}{x-1} < 0$ and, therefore, less than x (C is false).

11. **960** If s is the number of students enrolled, $\frac{1}{12}s$ is the number who were absent, and $\frac{11}{12}s$ is the number who were present. Since $\frac{1}{5}$ of them went on a field trip, $\frac{4}{5}$ of them stayed in school. Therefore,

$$704 = \frac{\cancel{4}^{1}}{5} \times \frac{11}{\cancel{12}_{3}}s = \frac{11}{15}s \Rightarrow$$

$$s = 704 \div \frac{11}{15} = 704 \times \frac{15}{11} = 960.$$

12. **(B)** Since $a < 1$, $\sqrt{a} > a$ (A is false). Since $a < 1$, $a^2 < a$ (B is true). The reciprocal of a positive number less than 1 is greater than 1 (C is false).

13. **(D)** There are two easy ways to do this. The first is to see that (34.56)(7.89) has 4 decimal places, whereas (.3456)(78.9) has 5, so the numerator has to be divided by 10. The second is to round off and calculate mentally: since $30 \times 8 = 240$, and $.3 \times 80 = 24$, we must divide by 10.

14. **(A)** Nine fractions are formed:

$$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{2}{2}, \frac{2}{3}, \frac{2}{4}, \frac{3}{2}, \frac{3}{3}, \frac{3}{4}$$

Note that although some of these fractions are equivalent, we do have nine distinct fractions.
When you multiply, the three 2s and the three 3s in the numerators cancel with the three 2s and three 3s in the denominators. So, the numerator is 1 and the denominator is $4 \times 4 \times 4 = 64$.

15. **(C)** Multiplying Ezra's incorrect answer by 1000 would undo the final division he made. At that point he should have multiplied by 1000. So, to correct his error, he should multiply again by 1000. In all, Ezra should multiply his incorrect answer by $1000 \times 1000 = 1{,}000{,}000$.

16. **(C)** Each quantity equals $\frac{5 \times 47}{3}$.

17. **(A)** Quantity A: $-\frac{2}{3} \times \frac{3}{5} = -\frac{2}{5}$.

Quantity B: $-\frac{2}{3} \div \frac{3}{5} = -\frac{2}{3} \times \frac{5}{3} = -\frac{10}{9}$.

Finally, $\frac{10}{9} > \frac{2}{5} \Rightarrow -\frac{10}{9} < -\frac{2}{5}$.

18. **(A)** Quantity A: $\dfrac{15}{\frac{1}{15}} = 15 \times 15 = 225$.

19. **(C)** $8 \div \frac{3}{4} = 8 \times \frac{4}{3} = \frac{32}{3} = 10\frac{2}{3}$. Since 10 packages wouldn't be enough, she had to buy 11. (10 packages would weigh only $7\frac{1}{2}$ pounds.)

20. **(B)** You don't need to multiply on this one: since $\frac{11}{12} < 1$, $\frac{11}{12}$ of $\frac{13}{14}$ is less than $\frac{13}{14}$, which is already less than $\frac{14}{15}$.

21. **(C)** Quantity B is the sum of 2 complex fractions:

$$\frac{\frac{1}{2}}{\frac{2}{3}} + \frac{\frac{2}{3}}{\frac{1}{2}}$$

Simplifying each complex fraction, by multiplying numerator and denominator by 6, or treating these as the quotient of 2 fractions, we get $\frac{3}{4} + \frac{4}{3}$, which is exactly the value of Quantity A.

22. **(A)** When two fractions have the same numerator, the one with the smaller denominator is bigger, and $2^{100} < 3^{100}$.

23. **(A)** Since Quantity A is the product of 4 negative numbers, it is positive, and so is greater than Quantity B, which, being the product of 3 negative numbers, is negative.

24. **(A)** Quantity A: $\frac{1}{2} \div \frac{1}{3} = \frac{1}{2} \times \frac{3}{1} = \frac{3}{2}$.

Since Quantity B is the reciprocal of Quantity A, Quantity B = $\frac{2}{3}$.

25. **(B)** If $0 < x < 1$, then $x^2 < x < \sqrt{x}$. In this question, $x = \frac{3}{11}$.

11-C. PERCENTS

The word ***percent*** means hundredth. We use the symbol "%" to express the word "percent." For example, "17 percent" means "17 hundredths," and can be written with a % symbol, as a fraction, or as a decimal:

$$17\% = \frac{17}{100} = 0.17.$$

TIP

A percent is just a fraction whose denominator is 100.

KEY FACT C1

- **To convert a percent to a decimal, drop the % symbol and move the decimal point two places to the left, adding 0s if necessary. (Remember that we assume that there is a decimal point to the right of any whole number.)**
- **To convert a percent to a fraction, drop the % symbol, write the number over 100, and reduce.**

$25\% = 0.25 = \frac{25}{100} = \frac{1}{4}$ $\quad 100\% = 1.00 = \frac{100}{100}$ $\quad 12.5\% = 0.125 = \frac{12.5}{100} = \frac{125}{1000} = \frac{1}{8}$

$1\% = 0.01 = \frac{1}{100}$ $\quad \frac{1}{2}\% = 0.5\% = 0.005 = \frac{.5}{100} = \frac{1}{200}$ $\quad 250\% = 2.50 = \frac{250}{100} = \frac{5}{2}$

KEY FACT C2

- **To convert a decimal to a percent, move the decimal point two places to the right, adding 0s if necessary, and add the % symbol.**
- **To convert a fraction to a percent, first convert the fraction to a decimal, then convert the decimal to a percent, as indicated above.**

$0.375 = 37.5\%$ $\quad 0.3 = 30\%$ $\quad 1.25 = 125\%$ $\quad 10 = 1000\%$

$\frac{3}{4} = 0.75 = 75\%$ $\quad \frac{1}{3} = 0.33333\ldots = 33.333\ldots\% = 33\frac{1}{3}\%$ $\quad \frac{1}{5} = 0.2 = 20\%$

You should be familiar with the following basic conversions:

$\frac{1}{2} = 50\%$	$\frac{1}{10} = 10\%$	$\frac{6}{10} = \frac{3}{5} = 60\%$
$\frac{1}{3} = 33\frac{1}{3}\%$	$\frac{2}{10} = \frac{1}{5} = 20\%$	$\frac{7}{10} = 70\%$
$\frac{2}{3} = 66\frac{2}{3}\%$	$\frac{3}{10} = 30\%$	$\frac{8}{10} = \frac{4}{5} = 80\%$
$\frac{1}{4} = 25\%$	$\frac{4}{10} = \frac{2}{5} = 40\%$	$\frac{9}{10} = 90\%$
$\frac{3}{4} = 75\%$	$\frac{5}{10} = \frac{1}{2} = 50\%$	$\frac{10}{10} = 1 = 100\%$

Knowing these conversions can help solve many problems more quickly. For example, the fastest way to find 25% of 32 is not to multiply 32 by 0.25; rather, it is to know that $25\% = \frac{1}{4}$, and that $\frac{1}{4}$ of 32 is 8.

Many questions involving percents can actually be answered more quickly in your head than by using paper and pencil. Since $10\% = \frac{1}{10}$, to take 10% of a number, just divide by 10 by moving the decimal point one place to the left: 10% of 60 is 6. Also, since 5% is half of 10%, then 5% of 60 is 3 (half of 6); and since 30% is 3 times 10%, then 30% of 60 is 18 (3×6).

Practice doing this, because improving your ability to do mental math will add valuable points to your score on the GRE.

CAUTION

Do not confuse 0.5 and 0.5%.
Just as 5 is 100 times 5%, 0.5 is 100 times 0.5% = 0.005.

CAUTION

Although 35% can be written as $\frac{35}{100}$ or 0.35, x% can *only* be written as $\frac{x}{100}$.

Solving Percent Problems

Consider the following three questions:

(i) What is 45% of 200?
(ii) 90 is 45% of what number?
(iii) 90 is what percent of 200?

The arithmetic needed to answer each of these questions is very easy, but unless you set a question up properly, you won't know whether you should multiply or divide. In each case, there is one unknown, which we will call x. Now just translate each sentence, replacing "is" by "=" and the unknown by x.

(i) $x = 45\%$ of $200 \Rightarrow x = .45 \times 200 = 90$
(ii) $90 = 45\%$ of $x \Rightarrow 90 = .45x \Rightarrow x = 90 \div .45 = 200$
(iii) $90 = x\%$ of $200 \Rightarrow 90 = \frac{x}{\cancel{100}_1}(\cancel{200}^2) \Rightarrow 90 = 2x \Rightarrow x = 45$

EXAMPLE 1

Charlie gave 20% of his baseball cards to Kenne and 15% to Paulie. If he still had 520 cards, how many did he have originally?

SOLUTION.
Originally, Charlie had 100% of the cards (all of them). Since he gave away 35% of them, he has 100% – 35% = 65% of them left. So, 520 is 65% of what number?

$$520 = .65x \Rightarrow x = 520 \div .65 = \mathbf{800}.$$

EXAMPLE 2

After Ruth gave 110 baseball cards to Alison and 75 to Susanna, she still had 315 left. What percent of her cards did Ruth give away?

Ⓐ 25% Ⓑ $33\frac{1}{3}$% Ⓒ 37% Ⓓ 40% Ⓔ 50%

SOLUTION. Ruth gave away a total of 185 cards and had 315 left. Therefore, she started with 185 + 315 = 500 cards. So, 185 is what percent of 500?

$$185 = \frac{x}{\cancel{100}_1}(\overset{5}{\cancel{500}}) \Rightarrow 5x = 185 \Rightarrow x = 185 \div 5 = \mathbf{37}$$

Ruth gave away 37% of her cards, **(C)**.

Since percent means hundredth, the easiest number to use in any percent problem is 100:

$$a\% \text{ of } 100 = \frac{a}{\cancel{100}_1} \times \overset{1}{\cancel{100}} = a.$$

KEY FACT C3

For any positive number *a*: *a*% of 100 is *a*.

For example: 91.2% of 100 is 91.2; 300% of 100 is 300; and $\frac{1}{2}$% of 100 is $\frac{1}{2}$.

TACTIC

C1

In any problem involving percents, use the number 100. (It doesn't matter whether or not 100 is a realistic number—a country can have a population of 100; an apple can cost $100; a man can run 100 miles per hour.)

EXAMPLE 3

In 1985 the populations of town A and town B were the same. From 1985 to 1995 the population of town A increased by 60% while the population of town B decreased by 60%. In 1995, the population of town B was what percent of the population of town A?

Ⓐ 25% Ⓑ 36% Ⓒ 40% Ⓓ 60% Ⓔ 120%

SOLUTION.
On the GRE, do not waste time with a nice algebraic solution. Simply, assume that in 1985 the population of each town was 100. Then, since 60% of 100 is 60, in 1995, the populations were 100 + 60 = 160 and 100 – 60 = 40. So, in 1995, town B's population was $\frac{40}{160} = \frac{1}{4} = \mathbf{25\%}$ of town A's **(A)**.

Since a% of b is $\frac{a}{100} \times b = \frac{ab}{100}$, and b% of a is $\frac{b}{100} \times a = \frac{ba}{100}$, we have the result shown in KEY FACT C4.

KEY FACT C4

For any positive numbers *a* and *b*: *a*% of *b* = *b*% of *a*.

KEY FACT C4 often comes up on the GRE in quantitative comparison questions: Which is greater, 13% of 87 or 87% of 13? Don't multiply — they're equal.

Percent Increase and Decrease

KEY FACT C5

- **The *percent increase* of a quantity is**
  $$\frac{\text{actual increase}}{\text{original amount}} \times 100\%.$$
- **The *percent decrease* of a quantity is**
  $$\frac{\text{actual decrease}}{\text{original amount}} \times 100\%.$$

For example:

- If the price of a lamp goes from $80 to $100, the actual increase is $20, and the percent increase is $\frac{20}{80} \times 100\% = \frac{1}{4} \times 100\% = 25\%$.
- If a $100 lamp is on sale for $80, the actual decrease in price is $20, and the percent decrease is $\frac{20}{100} \times 100\% = 20\%$.

Notice that the percent increase in going from 80 to 100 is not the same as the percent decrease in going from 100 to 80.

KEY FACT C6

If $a < b$, the percent increase in going from *a* to *b* is *always* greater than the percent decrease in going from *b* to *a*.

KEY FACT C7

- **To increase a number by $k\%$, multiply it by $(1 + k\%)$.**
- **To decrease a number by $k\%$, multiply it by $(1 - k\%)$.**

For example:

- The value of a \$1600 investment after a 25% increase is \$1600(1 + 25%) = \$1600(1.25) = \$2000.
- If the investment then loses 25% of its value, it is worth \$2000(1 – 25%) = \$2000(.75) = \$1500.

Note that, after a 25% increase followed by a 25% decrease, the value is \$1500, \$100 *less* than the original amount.

KEY FACT C8

An increase of $k\%$ followed by a decrease of $k\%$ is equal to a decrease of $k\%$ followed by an increase of $k\%$, and is *always* less than the original value. The original value is never regained.

EXAMPLE 4

Store B always sells CDs at 60% off the list price. Store A sells its CDs at 40% off the list price, but often runs a special sale during which it reduces its prices by 20%.

Quantity A	Quantity B
The price of a CD when it is on sale at store A	The price of the same CD at store B

SOLUTION.
Assume the list price of the CD is \$100. Store B always sells the CD for \$40 (\$60 off the list price). Store A normally sells the CD for \$60 (\$40 off the list price), but on sale reduces its price by 20%. Since 20% of 60 is 12, the sale price is \$48 (\$60 – \$12). The price is greater at Store A.

Notice that a decrease of 40% followed by a decrease of 20% is not the same as a single decrease of 60%; it is less. In fact, a decrease of 40% followed by a decrease of 30% wouldn't even be as much as a single decrease of 60%.

KEY FACT C9

- **A decrease of $a\%$ followed by a decrease of $b\%$ *always* results in a smaller decrease than a single decrease of $(a + b)\%$.**
- **An increase of $a\%$ followed by an increase of $b\%$ *always* results in a larger increase than a single increase of $(a + b)\%$.**
- **An increase (or decrease) of $a\%$ followed by another increase (or decrease) of $a\%$ is *never* the same as a single increase (or decrease) of $2a\%$.**

EXAMPLE 5

Sally and Heidi were both hired in January at the same salary. Sally got two 40% raises, one in July and another in November. Heidi got one 90% raise in October.

Quantity A	Quantity B
Sally's salary at the end of the year	Heidi's salary at the end of the year

SOLUTION.
Since this is a percent problem, assume their salaries were \$100. Quantity A: Sally's salary rose to 100(1.40) = 140, and then to 140(1.40) = \$196. Quantity B: Heidi's salary rose to 100(1.90) = \$190. Quantity **A** is greater.

EXAMPLE 6

In January, the value of a stock increased by 25%, and in February, it decreased by 20%. How did the value of the stock at the end of February compare with its value at the beginning of January?

Ⓐ It was less.
Ⓑ It was the same.
Ⓒ It was 5% greater.
Ⓓ It was more than 5% greater.
Ⓔ It cannot be determined from the information given.

SOLUTION. Assume that at the beginning of January the stock was worth \$100. Then at the end of January it was worth \$125. Since 20% of 125 is 25, during February its value decreased from \$125 to \$100. The answer is **B**.

KEY FACT C10

- **If a number is the result of increasing another number by $k\%$, to find the original number, divide by $(1 + k\%)$.**
- **If a number is the result of decreasing another number by $k\%$, to find the original number, divide it by $(1 - k\%)$.**

For example, if the population of a town in 1990 was 3000, and this represents an increase of 20% since 1980, to find the population in 1980, divide 3000 by (1 + 20%): 3000 ÷ 1.20 = 2500.

EXAMPLE 7

From 1989 to 1990, the number of applicants to a college increased 15% to 5060. How many applicants were there in 1989?

[ ]

SOLUTION.
The number of applicants in 1989 was 5060 ÷ 1.15 = **4400**.

CAUTION

Percents over 100%, which come up most often on questions involving percent increases, are often confusing for students. First of all, be sure you understand that 100% of a number is that number, 200% of a number is 2 times the number, and 1000% of a number is 10 times the number. If the value of an investment goes from $1000 to $5000, it is now worth 5 times, or 500%, as much as it was originally; but there has only been a *400%* increase in value:

$$\frac{\text{actual increase}}{\text{original amount}} \times 100\% = \frac{4000}{1000} \times 100\% = 4 \times 100\% = 400\%.$$

EXAMPLE 8

The population of a country doubled every 10 years from 1960 to 1990. What was the percent increase in population during this time?

Ⓐ 200% Ⓑ 300% Ⓒ 700% Ⓓ 800% Ⓔ 1000%

SOLUTION.
The population doubled three times (once from 1960 to 1970, again from 1970 to 1980, and a third time from 1980 to 1990). Assume that the population was originally 100. Then it increased from 100 to 200 to 400 to 800. So the population in 1990 was 8 times the population in 1960, but this was an increase of 700 people, or **700%** (**C**).

Practice Exercises — Percents

Discrete Quantitative Questions

1. If 25 students took an exam and 4 of them failed, what percent of them passed?
   Ⓐ 4%
   Ⓑ 21%
   Ⓒ 42%
   Ⓓ 84%
   Ⓔ 96%

2. Amanda bought a $60 sweater on sale at 5% off. How much did she pay, including 5% sales tax?
   Ⓐ $54.15
   Ⓑ $57.00
   Ⓒ $57.75
   Ⓓ $59.85
   Ⓔ $60.00

3. What is 10% of 20% of 30%?
   Ⓐ 0.006%
   Ⓑ 0.6%
   Ⓒ 6%
   Ⓓ 60%
   Ⓔ 6000%

4. If c is a positive number, 500% of c is what percent of $500c$?
   Ⓐ 0.01
   Ⓑ 0.1
   Ⓒ 1
   Ⓓ 10
   Ⓔ 100

5. What percent of 50 is b?
   Ⓐ $\frac{b}{50}$
   Ⓑ $\frac{b}{2}$
   Ⓒ $\frac{50}{b}$
   Ⓓ $\frac{2}{b}$
   Ⓔ $2b$

6. 8 is $\frac{1}{3}$% of what number?

   [ ]

7. During his second week on the job, Mario earned $110. This represented a 25% increase over his earnings of the previous week. How much did he earn during his first week of work?
   Ⓐ $82.50
   Ⓑ $85.00
   Ⓒ $88.00
   Ⓓ $137.50
   Ⓔ $146.67

8. At Bernie's Bargain Basement everything is sold for 20% less than the price marked. If Bernie buys radios for $80, what price should he mark them if he wants to make a 20% profit on his cost?
   Ⓐ $96
   Ⓑ $100
   Ⓒ $112
   Ⓓ $120
   Ⓔ $125

9. Mrs. Fisher usually deposits the same amount of money each month into a vacation fund. This year she decided not to make any contributions during November and December. To make the same annual contribution that she had originally planned, by what percent should she increase her monthly deposits from January through October?

Ⓐ $16\frac{2}{3}$%
Ⓑ 20%
Ⓒ 25%
Ⓓ $33\frac{1}{3}$%
Ⓔ It cannot be determined from the information given.

10. The price of a loaf of bread was increased by 20%. How many loaves can be purchased for the amount of money that used to buy 300 loaves?

Ⓐ 240
Ⓑ 250
Ⓒ 280
Ⓓ 320
Ⓔ 360

11. If 1 micron = 10,000 angstroms, then 100 angstroms is what percent of 10 microns?

Ⓐ 0.0001%
Ⓑ 0.001%
Ⓒ 0.01%
Ⓓ 0.1%
Ⓔ 1%

12. There are twice as many girls as boys in an English class. If 30% of the girls and 45% of the boys have already handed in their book reports, what percent of the students have not yet handed in their reports?

[ ] %

13. An art dealer bought a Ming vase for $1000 and later sold it for $10,000. By what percent did the value of the vase increase?

Ⓐ 10%
Ⓑ 90%
Ⓒ 100%
Ⓓ 900%
Ⓔ 1000%

14. During a sale a clerk was putting a new price tag on each item. On one jacket, he accidentally raised the price by 15% instead of lowering the price by 15%. As a result the price on the tag was $45 too high. What was the original price of the jacket?

[ ] dollars

15. On a test consisting of 80 questions, Eve answered 75% of the first 60 questions correctly. What percent of the other 20 questions does she need to answer correctly for her grade on the entire exam to be 80%?

Ⓐ 85%
Ⓑ 87.5%
Ⓒ 90%
Ⓓ 95%
Ⓔ 100%

Quantitative Comparison Questions

Ⓐ Quantity A is greater.
Ⓑ Quantity B is greater.
Ⓒ Quantities A and B are equal.
Ⓓ It is impossible to determine which quantity is greater.

	Quantity A	Quantity B
16.	400% of 3	300% of 4

n% of 25 is 50

	Quantity A	Quantity B
17.	50% of n	75

	Quantity A	Quantity B
18.	The price of a television when it is on sale at 25% off	The price of that television when it's on sale at $25 off

The price of cellular phone 1 is 20% more than the price of cellular phone 2.

	Quantity A	Quantity B
19.	The price of cellular phone 1 when it is on sale at 20% off	The price of cellular phone 2

	Quantity A	Quantity B
20.	$\frac{2}{3}$% of $\frac{3}{4}$	$\frac{3}{4}$% of $\frac{2}{3}$

	Quantity A	Quantity B
21.	a% of $\frac{1}{b}$	b% of $\frac{1}{a}$

Bank A pays 5% interest on its savings accounts.
Bank B pays 4% interest on its savings accounts.

	Quantity A	Quantity B
22.	Percent by which bank B would have to raise its interest rate to match bank A	20%

A solution that is 20% sugar is made sweeter by doubling the amount of sugar.

	Quantity A	Quantity B
23.	The percent of sugar in the new solution	40%

b is an integer greater than 1, and b equals n% of b^2

	Quantity A	Quantity B
24.	n	50

After Ali gave Lior 50% of her money, she had 20% as much as he did.

	Quantity A	Quantity B
25.	75% of the amount Lior had originally	150% of the amount Ali had originally

ANSWER KEY

1. **D**	6. **2400**	11. **D**	16. **C**	21. **D**
2. **D**	7. **C**	12. **65**	17. **A**	22. **A**
3. **B**	8. **D**	13. **D**	18. **D**	23. **B**
4. **C**	9. **B**	14. **150**	19. **B**	24. **D**
5. **E**	10. **B**	15. **D**	20. **C**	25. **C**

Answer Explanations

1. **(D)** If 4 students failed, then the other 25 – 4 = 21 students passed, and $\frac{21}{25} = 0.84 = 84\%$.

2. **(D)** Since 5% of 60 is 3, Amanda saved \$3, and thus paid \$57 for the sweater. She then had to pay 5% sales tax on the \$57: $.05 \times 57 = 2.85$, so the total cost was \$57 + \$2.85 = \$59.85.

3. **(B)** 10% of 20% of 30% = $.10 \times .20 \times .30 = .006 = .6\%$.

4. **(C)** 500% of $c = 5c$, which is 1% of $500c$.

5. **(E)** $b = \frac{x}{100}(50) \Rightarrow b = \frac{x}{2} \Rightarrow x = 2b$.

6. **2400** $8 = \frac{\frac{1}{3}}{100}x = \frac{1}{300}x \Rightarrow x = 8 \times 300 = 2400$.

7. **(C)** To find Mario's earnings during his first week, divide his earnings from the second week by 1.25: $110 \div 1.25 = 88$.

8. **(D)** Since 20% of 80 is 16, Bernie wants to get \$96 for each radio he sells. What price should the radios be marked so that after a 20% discount, the customer will pay \$96? If x represents the marked price, then $.80x = 96 \Rightarrow x = 96 \div .80 = 120$.

9. **(B)** Assume that Mrs. Fisher usually contributed \$100 each month, for an annual total of \$1200. Having decided not to contribute for 2 months, the \$1200 will have to be paid in 10 monthly deposits of \$120 each. This is an increase of \$20, and a percent increase of

$$\frac{\text{actual increase}}{\text{original amount}} \times 100\% = \frac{20}{100} \times 100\% = 20\%.$$

10. **(B)** Assume that a loaf of bread used to cost \$1 and that now it costs \$1.20 (20% more). Then 300 loaves of bread used to cost \$300. How many loaves costing \$1.20 each can be bought for \$300? $300 \div 1.20 = 250$.

11. **(D)** 1 micron = 10,000 angstroms ⇒ 10 microns = 100,000 angstroms; dividing both sides by 1000, we get 100 angstroms = $\frac{1}{1000}$ (10 microns); and $\frac{1}{1000} = .001 = 0.1\%$.

12. **65** Assume that there are 100 boys and 200 girls in the class. Then, 45 boys and 60 girls have handed in their reports. So 105 students have handed them in, and 300 – 105 = 195 have not handed them in. What percent of 300 is 195?

$$\frac{195}{300} = .65 = 65\%.$$

13. **(D)** The increase in the value of the vase was \$9,000. So the percent increase is $\frac{\text{actual increase}}{\text{original cost}} \times 100\% = \frac{9000}{1000} = 9 = 900\%$.

14. **150** If p represents the original price, the jacket was priced at $1.15p$ instead of $.85p$. Since this was a \$45 difference, $45 = 1.15p - .85p = 30p \Rightarrow p = 45 \div .30 = \$150$.

15. **(D)** To earn a grade of 80% on the entire exam, Eve needs to correctly answer 64 questions (80% of 80). So far, she has answered 45 questions correctly (75% of 60). Therefore, on the last 20 questions she needs 64 – 45 = 19 correct answers; and $\frac{19}{20} = 95\%$.

16. **(C)** Quantity A: 400% of 3 = $4 \times 3 = 12$.
Quantity B: 300% of 4 = $3 \times 4 = 12$.

17. **(A)** Since n% of 25 is 50, then 25% of n is also 50, and 50% of n is twice as much: 100. If you don't see that, just solve for n:

$$\frac{n}{100} \times 25 = 50 \Rightarrow \frac{n}{4} = 50 \Rightarrow n = 200 \text{ and } 50\% \text{ of } n = 100.$$

18. **(D)** A 25% discount on a \$10 television is much less than \$25, whereas a 25% discount on a \$1000 television is much more than \$25. (They would be equal only if the regular price of the television were \$100.)

19. **(B)** Assume that the list price of cellular phone 2 is \$100; then the list price of cellular phone 1 is \$120, and on sale at 20% off it costs \$24 less: \$96.

20. **(C)** For *any* numbers a and b: a% of b is equal to b% of a.

21. **(D)**

Quantity A	Quantity B
$a\%$ of $\frac{1}{b}$	$b\%$ of $\frac{1}{a}$
$\frac{a}{100} \times \frac{1}{b} = \frac{a}{100b}$	$\frac{b}{100} \times \frac{1}{a} = \frac{b}{100a}$

Multiply by 100:

$\frac{a}{b}$	$\frac{b}{a}$

The quantities are equal if a and b are equal, and unequal otherwise.

22. **(A)** Bank B would have to increase its rate from 4% to 5%, an actual increase of 1%. This represents a percent increase of $\frac{1\%}{4\%} \times 100\% = 25\%$.

23. **(B)** Assume a vat contains 100 ounces of a solution, of which 20% or 20 ounces is sugar (the remaining 80 ounces being water). If the amount of sugar is doubled, there would be 40 ounces of sugar and 80 ounces of water. The sugar will then comprise $\frac{40}{120} = \frac{1}{3} = 33\frac{1}{3}\%$ of the solution.

24. **(D)** If $b = 2$, then $b^2 = 4$, and $2 = 50\%$ of 4; in this case, the quantities are equal. If $b = 4$, $b^2 = 16$, and 4 is not 50% of 16; in this case, the quantities are not equal.

25. **(C)** Avoid the algebra and just assume Ali started with \$100. After giving Lior \$50, she had \$50 left, which was 20% or one-fifth of what he had. So, Lior had $5 \times \$50 = \$250$, which means that originally he had \$200.
Quantity A: 75% of \$200 = \$150.
Quantity B: 150% of \$100 = \$150.
The quantities are equal.

11-D. RATIOS AND PROPORTIONS

A ***ratio*** is a fraction that compares two quantities that are measured in the same units. The first quantity is the numerator and the second quantity is the denominator.

For example, if there are 4 boys and 16 girls on the debate team, we say that the ratio of the number of boys to the number of girls on the team is 4 to 16, or $\frac{4}{16}$. This is often written 4:16. Since a ratio is just a fraction, it can be reduced or converted to a decimal or a percent. The following are all different ways to express the same ratio:

4 to 16 4:16 $\frac{4}{16}$ 2 to 8 2:8 $\frac{2}{8}$ 1 to 4 1:4 $\frac{1}{4}$ 0.25 25%

CAUTION

Saying that the ratio of boys to girls on the team is 1:4 does *not* mean that $\frac{1}{4}$ of the team members are boys. It means that for each boy on the team there are 4 girls; so for every 5 members of the team, there are 4 girls and 1 boy. Boys, therefore, make up $\frac{1}{5}$ of the team, and girls $\frac{4}{5}$.

KEY FACT D1

If a set of objects is divided into two groups in the ratio of a:b, then the first group contains $\frac{a}{a+b}$ of the objects and the second group contains $\frac{b}{a+b}$ of the objects.

EXAMPLE 1

Last year, the ratio of the number of tennis matches that Central College's women's team won to the number of matches they lost was 7:3. What percent of their matches did the team win?

[   ] %

SOLUTION.

The team won $\frac{7}{7+3} = \frac{7}{10} = $ **70%** of their matches.

EXAMPLE 2

If 45% of the students at a college are male, what is the ratio of male students to female students?

SOLUTION.
Assume that there are 100 students. Then 45 of them are male, and $100 - 45 = 55$ of them are female. So, the ratio of males to females is $\frac{45}{55} = \frac{9}{11}$.

TIP
In problems involving percents the best number to use is 100.

If we know how many boys and girls there are in a club, then, clearly, we know not only the ratio of boys to girls, but several other ratios too. For example, if the club has 7 boys and 3 girls: the ratio of boys to girls is $\frac{7}{3}$, the ratio of girls to boys is $\frac{3}{7}$, the ratio of boys to members is $\frac{7}{10}$, the ratio of members to girls is $\frac{10}{3}$, and so on.

However, if we know a ratio, we *cannot* determine how many objects there are. For example, if a jar contains only red and blue marbles, and if the ratio of red marbles to blue marbles is 3:5, there *may* be 3 red marbles and 5 blue marbles, but *not necessarily.* There may be 300 red marbles and 500 blue ones, since the ratio 300:500 reduces to 3:5. In the same way, all of the following are possibilities for the distribution of marbles.

Red	6	12	33	51	150	3000	$3x$
Blue	10	20	55	85	250	5000	$5x$

The important thing to observe is that the number of red marbles can be *any* multiple of 3, as long as the number of blue marbles is the *same* multiple of 5.

KEY FACT D2

If two numbers are in the ratio of $a:b$, then for some number x, the first number is ax and the second number is bx. If the ratio is in lowest terms, and if the quantities must be integers, then x is also an integer.

TACTIC D1

In any ratio problem, write the letter x after each number and use some given information to solve for x.

EXAMPLE 3

If the ratio of men to women in a particular dormitory is 5:3, which of the following could not be the number of residents in the dormitory?

Ⓐ 24 Ⓑ 40 Ⓒ 96 Ⓓ 150 Ⓔ 224

SOLUTION.
If $5x$ and $3x$ are the number of men and women in the dormitory, respectively, then the number of residents in the dormitory is $5x + 3x = 8x$. So, the number of students must be a multiple of 8. Of the five choices, only **150** **(D)** is not divisible by 8.

NOTE: Assume that the ratio of the number of pounds of cole slaw to the number of pounds of potato salad consumed in the dormitory's cafeteria was 5:3. Then, it is possible that a total of exactly 150 pounds was eaten: 93.75 pounds of cole slaw and 56.25 pounds of potato salad. In Example 3, 150 wasn't possible because there had to be a *whole* number of men and women.

EXAMPLE 4

The measures of the two acute angles in a right triangle are in the ratio of 5:13. What is the measure of the larger angle?

Ⓐ 25° Ⓑ 45° Ⓒ 60° Ⓓ 65° Ⓔ 75°

SOLUTION.
Let the measure of the smaller angle be $5x$ and the measure of the larger angle be $13x$. Since the sum of the measures of the two acute angles of a right triangle is 90° (KEY FACT J1), $5x + 13x = 90 \Rightarrow 18x = 90 \Rightarrow x = 5$.

Therefore, the measure of the larger angle is $13 \times 5 =$ **65°** **(D)**.

Ratios can be extended to three or four or more terms. For example, we can say that the ratio of freshmen to sophomores to juniors to seniors in a college marching band is 6:8:5:8, which means that for every 6 freshmen in the band there are 8 sophomores, 5 juniors, and 8 seniors.

TIP
TACTIC D1 applies to extended ratios, as well.

EXAMPLE 5

The concession stand at Cinema City sells popcorn in three sizes: large, super, and jumbo. One day, Cinema City sold 240 bags of popcorn, and the ratio of large to super to jumbo was 8:17:15. How many super bags of popcorn were sold that day?

SOLUTION.
Let $8x$, $17x$, and $15x$ be the number of large, super, and jumbo bags of popcorn sold, respectively. Then $8x + 17x + 15x = 240 \Rightarrow 40x = 240 \Rightarrow x = 6$.
The number of super bags sold was $17 \times 6 =$ **102**.

KEY FACT D3

KEY FACT D1 applies to extended ratios, as well. If a set of objects is divided into 3 groups in the ratio $a:b:c$, then the first group contains $\frac{a}{a+b+c}$ of the objects, the second $\frac{b}{a+b+c}$, and the third $\frac{c}{a+b+c}$.

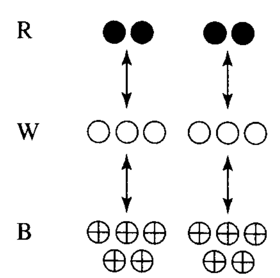
EXAMPLE 6

If the ratio of large to super to jumbo bags of popcorn sold at Cinema City was 8:17:15, what percent of the bags sold were super?

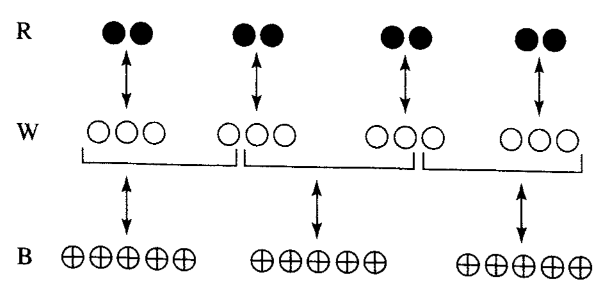
Ⓐ 20% Ⓑ 25% Ⓒ $33\frac{1}{3}$% Ⓓ 37.5% Ⓔ 42.5%

SOLUTION.
Super bags made up $\frac{17}{8+17+15} = \frac{17}{40} =$ **42.5%** of the total (**E**).

A jar contains a number of red (R), white (W), and blue (B) marbles. Suppose that R:W = 2:3 and W:B = 3:5. Then, for every 2 red marbles, there are 3 white ones, and for those 3 white ones, there are 5 blue ones. So, R:B = 2:5, and we can form the extended ratio R:W:B = 2:3:5.



If the ratios were R:W = 2:3 and W:B = 4:5, however, we wouldn't be able to combine them as easily. From the diagram below, you see that for every 8 reds there are 15 blues, so R:B = 8:15.



To see this without drawing a picture, we write the ratios as fractions: $\frac{R}{W} = \frac{2}{3}$ and $\frac{W}{B} = \frac{4}{5}$. Then, we multiply the fractions:

$$\frac{R}{\not{W}} \times \frac{\not{W}}{B} = \frac{2}{3} \times \frac{4}{5} = \frac{8}{15}, \quad \text{so} \quad \frac{R}{B} = \frac{8}{15}.$$

Not only does this give us R:B = 8:15, but also, if we multiply both W numbers, $3 \times 4 = 12$, we can write the extended ratio: R:W:B = 8:12:15.

EXAMPLE 7

Jar A and jar B each have 70 marbles,
all of which are red, white, or blue.
In jar A, R:W = 2:3 and W:B = 3:5.
In jar B, R:W = 2:3 and W:B = 4:5.

Quantity A	Quantity B
The number of white marbles in jar A	The number of white marbles in jar B

SOLUTION.
From the discussion immediately preceding this example, in jar A the extended ratio R:W:B is 2:3:5, which implies that the white marbles constitute $\frac{3}{2+3+5} = \frac{3}{10}$ of the total: $\frac{3}{10} \times 70 =$ **21**.

In jar B the extended ratio R:W:B is 8:12:15, so the white marbles are $\frac{12}{8+12+15} = \frac{12}{35}$ of the total: $\frac{12}{35} \times 70 =$ **24**. The answer is **B**.

A ***proportion*** is an equation that states that two ratios are equivalent. Since ratios are just fractions, any equation such as $\frac{4}{6} = \frac{10}{15}$ in which each side is a single fraction is a proportion. Usually the proportions you encounter on the GRE involve one or more variables.

TACTIC

D2

Solve proportions by cross-multiplying: if $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.

Setting up a proportion is a common way of solving a problem on the GRE.

EXAMPLE 8

If $\frac{3}{7} = \frac{x}{84}$, what is the value of x?

SOLUTION.
Cross-multiply: $3(84) = 7x \Rightarrow 252 = 7x \Rightarrow x =$ **36**.

EXAMPLE 9

If $\frac{x+2}{17} = \frac{x}{16}$, what is the value of $\frac{x+6}{19}$?

Ⓐ $\frac{1}{2}$ Ⓑ 1 Ⓒ $\frac{3}{2}$ Ⓓ 2 Ⓔ 3

SOLUTION.
Cross-multiply: $16(x + 2) = 17x \Rightarrow 16x + 32 = 17x \Rightarrow x = 32$.

So, $\frac{x+6}{19} = \frac{32+6}{19} = \frac{38}{19} =$ **2 (D)**.

EXAMPLE 10

A state law requires that on any field trip the ratio of the number of chaperones to the number of students must be at least 1:12. If 100 students are going on a field trip, what is the minimum number of chaperones required?

Ⓐ 6 Ⓑ 8 Ⓒ $8\frac{1}{3}$ Ⓓ 9 Ⓔ 12

SOLUTION.
Let x represent the number of chaperones required, and set up a proportion: $\frac{\text{number of chaperones}}{\text{number of students}} = \frac{1}{12} = \frac{x}{100}$. Cross-multiply: $100 = 12x \Rightarrow x = 8\frac{1}{3}$. This, of course, is *not* the answer since, clearly, the number of chaperones must be a whole number. Since x is greater than 8, 8 chaperones would not be enough. The answer is **9 (D)**.

A ***rate*** is a fraction that compares two quantities measured in different units. The word "per" often appears in rate problems: miles per hour, dollars per week, cents per ounce, students per classroom, and so on.

TIP

A rate can always be written as a fraction.

TACTIC

D3

Set up rate problems just like ratio problems. Solve the proportions by cross-multiplying.

EXAMPLE 11

Brigitte solved 24 math problems in 15 minutes. At this rate, how many problems can she solve in 40 minutes?

Ⓐ 25 Ⓑ 40 Ⓒ 48 Ⓓ 60 Ⓔ 64

SOLUTION.
Handle this rate problem exactly like a ratio problem. Set up a proportion and cross-multiply:

$$\frac{\text{problems}}{\text{minutes}} = \frac{24}{15} = \frac{x}{40} \Rightarrow 15x = 40 \times 24 = 960 \Rightarrow x = \mathbf{64}\ \mathbf{(E)}.$$

When the denominator in the given rate is 1 unit (1 minute, 1 mile, 1 dollar), the problem can be solved by a single division or multiplication. Consider Examples 12 and 13.

EXAMPLE 12

If Stefano types at the rate of 35 words per minute, how long will it take him to type 987 words?

SOLUTION.
Set up a proportion and cross-multiply:

$$\frac{\text{words typed}}{\text{minutes}} = \frac{35}{1} = \frac{987}{x} \Rightarrow 35x = 987 \Rightarrow x = \frac{987}{35} = \mathbf{28.2}\text{ minutes.}$$

EXAMPLE 13

If Mario types at the rate of 35 words per minute, how many words can he type in 85 minutes?

SOLUTION.
Set up a proportion and cross-multiply:

$$\frac{\text{words typed}}{\text{minutes}} = \frac{35}{1} = \frac{x}{85} \Rightarrow x = 35 \times 85 = \mathbf{2975} \text{ words.}$$

Notice that in Example 12, all we did was divide 987 by 35, and in Example 13, we multiplied 35 by 85. If you realize that, you don't have to introduce x and set up a proportion. You must know, however, whether to multiply or divide. If you're not absolutely positive which is correct, write the proportion; then you can't go wrong.

CAUTION

In rate problems it is essential that the units in both fractions be the same.

EXAMPLE 14

If 3 apples cost 50¢, how many apples can you buy for $20?

Ⓐ 20 Ⓑ 60 Ⓒ 120 Ⓓ 600 Ⓔ 2000

SOLUTION.
We have to set up a proportion, but it is *not* $\frac{3}{50} = \frac{x}{20}$. In the first fraction, the denominator represents *cents*, whereas in the second fraction, the denominator represents *dollars*. The units must be the same. We can change 50 cents to 0.5 dollar or we can change 20 dollars to 2000 cents:

$$\frac{3}{50} = \frac{x}{2000} \Rightarrow 50x = 6000 \Rightarrow x = \mathbf{120} \text{ apples } \mathbf{(C)}.$$

On the GRE, some rate problems involve only variables. They are handled in exactly the same way.

EXAMPLE 15

If a apples cost c cents, how many apples can be bought for d dollars?

Ⓐ $100acd$ Ⓑ $\frac{100d}{ac}$ Ⓒ $\frac{ad}{100c}$ Ⓓ $\frac{c}{100ad}$ Ⓔ $\frac{100ad}{c}$

SOLUTION.
First change d dollars to $100d$ cents, and set up a proportion: $\frac{\text{apples}}{\text{cents}} = \frac{a}{c} = \frac{x}{100d}$.

Now cross-multiply: $100ad = cx \Rightarrow x = \frac{100ad}{c}$ **(E)**.

Most students find problems such as Example 15 very difficult. If you get stuck on such a problem, use TACTIC 2, Chapter 8, which gives another strategy for handling these problems.

Notice that in rate problems, as one quantity increases or decreases, so does the other. If you are driving at 45 miles per hour, the more hours you drive, the further you go; if you drive fewer miles, it takes less time. If chopped meat cost $3.00 per pound, the less you spend, the fewer pounds you get; the more meat you buy, the more it costs.

In some problems, however, as one quantity increases, the other decreases. These *cannot* be solved by setting up a proportion. Consider the following two examples, which look similar but must be handled differently.

EXAMPLE 16

A hospital needs 150 pills to treat 6 patients for a week. How many pills does it need to treat 10 patients for a week?

SOLUTION.
Example 16 is a standard rate problem. The more patients there are, the more pills are needed.

The *ratio* or *quotient* remains constant: $\frac{150}{6} = \frac{x}{10} \Rightarrow 6x = 1500 \Rightarrow x = \mathbf{250}$.

EXAMPLE 17

A hospital has enough pills on hand to treat 10 patients for 14 days. How long will the pills last if there are 35 patients?

SOLUTION.
In Example 17, the situation is different. With more patients, the supply of pills will last for a shorter period of time; if there were fewer patients, the supply would last longer. It is not the ratio that remains constant, it is the *product*.

There are enough pills to last for 10 × 14 = 140 patient-days:

$$\frac{140 \text{ patient-days}}{10 \text{ patients}} = 14 \text{ days} \qquad \frac{140 \text{ patient-days}}{35 \text{ patients}} = \mathbf{4} \text{ days}$$

$$\frac{140 \text{ patient-days}}{70 \text{ patients}} = 2 \text{ days} \qquad \frac{140 \text{ patient-days}}{1 \text{ patient}} = 140 \text{ days}$$

There are many mathematical situations in which one quantity increases as another decreases, but their product is not constant. Those types of problems, however, do not appear on the GRE.

TACTIC
D4

If one quantity increases as a second quantity decreases, multiply them; their product will be a constant.

EXAMPLE 18

If 15 workers can pave a certain number of driveways in 24 days, how many days will 40 workers take, working at the same rate, to do the same job?

Ⓐ 6 Ⓑ 9 Ⓒ 15 Ⓓ 24 Ⓔ 40

SOLUTION.
Clearly, the more workers there are, the less time it will take, so use TACTIC D4: multiply. The job takes $15 \times 24 = 360$ worker-days:

$$\frac{360 \text{ worker-days}}{40 \text{ workers}} = \mathbf{9} \text{ days } (\mathbf{B}).$$

Note that it doesn't matter how many driveways have to be paved, as long as the 15 workers and the 40 workers are doing the same job. Even if the question had said, "15 workers can pave 18 driveways in 24 days," the number 18 would not have entered into the solution. This number would be important only if the second group of workers was going to pave a different number of driveways.

EXAMPLE 19

If 15 workers can pave 18 driveways in 24 days, how many days would it take 40 workers to pave 22 driveways?

Ⓐ 6 Ⓑ 9 Ⓒ 11 Ⓓ 15 Ⓔ 18

SOLUTION.
This question is similar to Example 18, except that now the jobs that the two groups of workers are doing are different. The solution, however, starts out exactly the same way. Just as in Example 18, 40 workers can do in 9 days the *same* job that 15 workers can do in 24 days. Since that job is to pave 18 driveways, 40 workers can pave $18 \div 9 = 2$ driveways every day. So, it will take **11** days for them to pave 22 driveways (**C**).

Practice Exercises—Ratios and Proportions

Discrete Quantitative Questions

1. If $\frac{3}{4}$ of the employees in a supermarket are not college graduates, what is the ratio of the number of college graduates to those who are not college graduates?

Ⓐ 1:3
Ⓑ 3:7
Ⓒ 3:4
Ⓓ 4:3
Ⓔ 3:1

2. If $\frac{a}{9} = \frac{10}{2a}$, what is the value of a^2?

Ⓐ $3\sqrt{6}$
Ⓑ $3\sqrt{5}$
Ⓒ $9\sqrt{6}$
Ⓓ 45
Ⓔ 90

3. If 80% of the applicants to a program were rejected, what is the ratio of the number accepted to the number rejected?

4. Scott can read 50 pages per hour. At this rate, how many pages can he read in 50 minutes?

Ⓐ 25
Ⓑ $41\frac{2}{3}$
Ⓒ $45\frac{1}{2}$
Ⓓ 48
Ⓔ 60

5. If all the members of a team are juniors or seniors, and if the ratio of juniors to seniors on the team is 3:5, what percent of the team members are seniors?

Ⓐ 37.5%
Ⓑ 40%
Ⓒ 60%
Ⓓ 62.5%
Ⓔ It cannot be determined from the information given.

6. The measures of the three angles in a triangle are in the ratio of 1:1:2. Which of the following statements must be true?

Indicate *all* such statements.

[A] The triangle is isosceles.
[B] The triangle is a right triangle.
[C] The triangle is equilateral.

7. What is the ratio of the circumference of a circle to its radius?

Ⓐ 1
Ⓑ $\frac{\pi}{2}$
Ⓒ $\sqrt{\pi}$
Ⓓ π
Ⓔ 2π

8. The ratio of the number of freshmen to sophomores to juniors to seniors on a college basketball team is 4:7:6:8. What percent of the team are sophomores?

Ⓐ 16%
Ⓑ 24%
Ⓒ 25%
Ⓓ 28%
Ⓔ 32%

9. At Central State College the ratio of the number of students taking Spanish to the number taking French is 7:2. If 140 students are taking French, how many are taking Spanish?

[ ] students

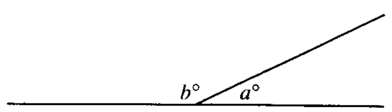
10. If $a:b = 3:5$ and $a:c = 5:7$, what is the value of $b:c$?

Ⓐ 3:7
Ⓑ 21:35
Ⓒ 21:25
Ⓓ 25:21
Ⓔ 7:3

11. If x is a positive number and $\frac{x}{3} = \frac{12}{x}$, then $x =$

Ⓐ 3
Ⓑ 4
Ⓒ 6
Ⓓ 12
Ⓔ 36

12. In the diagram below, $b:a = 7:2$. What is $b - a$?



Ⓐ 20
Ⓑ 70
Ⓒ 100
Ⓓ 110
Ⓔ 160

13. A snail can move i inches in m minutes. At this rate, how many feet can it move in h hours?

Ⓐ $\frac{5hi}{m}$
Ⓑ $\frac{60hi}{m}$
Ⓒ $\frac{hi}{12m}$
Ⓓ $\frac{5m}{hi}$
Ⓔ $5him$

14. Gilda can grade t tests in $\frac{1}{x}$ hours. At this rate, how many tests can she grade in x hours?

Ⓐ tx
Ⓑ tx^2
Ⓒ $\frac{1}{t}$
Ⓓ $\frac{x}{t}$
Ⓔ $\frac{1}{tx}$

15. A club had 3 boys and 5 girls. During a membership drive the same number of boys and girls joined the club. How many members does the club have now if the ratio of boys to girls is 3:4?

Ⓐ 12
Ⓑ 14
Ⓒ 16
Ⓓ 21
Ⓔ 28

16. If $\frac{3x-1}{25} = \frac{x+5}{11}$, what is the value of x?

Ⓐ $\frac{3}{4}$
Ⓑ 3
Ⓒ 7
Ⓓ 17
Ⓔ 136

17. If 4 boys can shovel a driveway in 2 hours, how many minutes will it take 5 boys to do the job?

Ⓐ 60
Ⓑ 72
Ⓒ 96
Ⓓ 120
Ⓔ 150

18. If 500 pounds of mush will feed 20 pigs for a week, for how many days will 200 pounds of mush feed 14 pigs?

[ ]

Quantitative Comparison Questions

Ⓐ Quantity A is greater.
Ⓑ Quantity B is greater.
Ⓒ Quantities A and B are equal.
Ⓓ It is impossible to determine which quantity is greater.

The ratio of red to blue marbles in a jar was 3:5. The same number of red and blue marbles were added to the jar.

	Quantity A	Quantity B
19.	The ratio of red to blue marbles now	3:5

Three associates agreed to split the $3000 profit of an investment in the ratio of 2:5:8.

	Quantity A	Quantity B
20.	The difference between the largest and the smallest share	$1200

The ratio of the number of boys to girls in the chess club is 5:2. The ratio of the number of boys to girls in the glee club is 11:4.

	Quantity A	Quantity B
21.	The number of boys in the chess club	The number of boys in the glee club

Sally invited the same number of boys and girls to her party. Everyone who was invited came, but 5 additional boys showed up. This caused the ratio of girls to boys at the party to be 4:5.

	Quantity A	Quantity B
22.	The number of people she invited to her party	40

A large jar is full of marbles. When a single marble is drawn at random from the jar, the probability that it is red is $\frac{3}{7}$.

	Quantity A	Quantity B
23.	The ratio of the number of red marbles to non-red marbles in the jar	$\frac{1}{2}$

$3a = 2b$ and $3b = 5c$

	Quantity A	Quantity B
24.	The ratio of a to c	1

The radius of circle II is 3 times the radius of circle I

	Quantity A	Quantity B
25.	$\frac{\text{area of circle II}}{\text{area of circle I}}$	3π